CS4311
Design and Analysis of Algorithms

Suffix Tree and Suffix Array

About this tutorial

- Introduce two data structures for text indexing problem:

  Suffix Tree and Suffix Array

Text Indexing

String Matching problem:

Given a text T and a pattern P, how to locate all occurrences of P in T ?

- KMP algorithm can solve this in O(|T|+|P|) time ➔ optimal

- In some applications, T is very long, and given in advance, and we will search different patterns against it later
  - E.g., T= Human DNA, P = gene

Text Indexing

Text Indexing problem:

Suppose a text T is known.

Can we build a data structure for T, such that for any pattern P given later, we can find all occurrences of P in T quickly ?

- The data structure is called an index of T
- Target: search better than $O(|T|+|P|)$??

Text Indexing

- Two main kinds of text indexes:

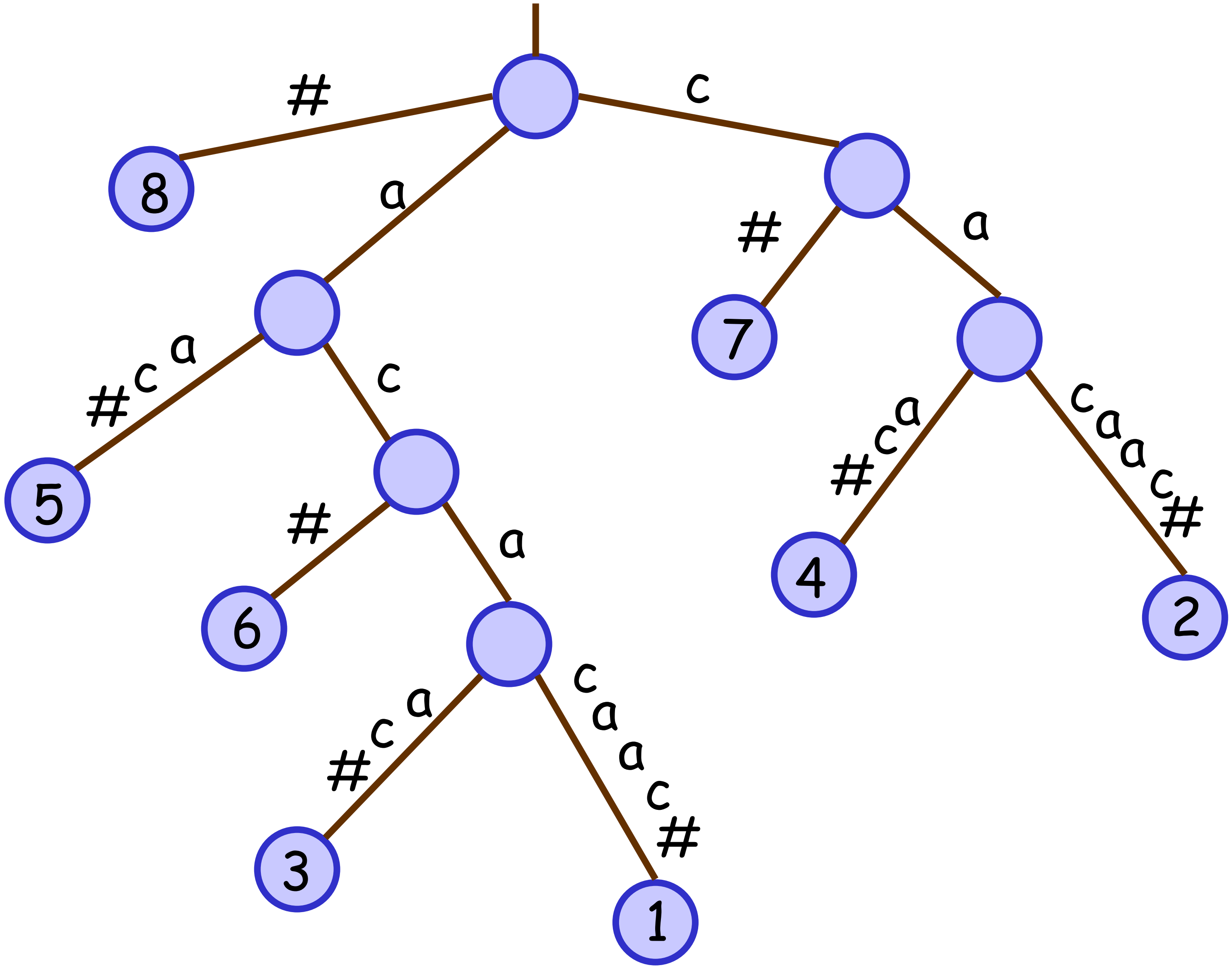
  Word-Based: (for texts formed by words)
  - Used by most text search engine
  - E.g., Inverted Files

  Full-Text: (for texts with no word boundaries)
  - Used in indexing DNA
  - E.g., Suffix Tree, Suffix Array

Suffix Tree

- Let T[1..n] be a text with n characters
  - we assume T[n] is a unique character
- For any j, T[j..n] is called a suffix of T

  → T has exactly n suffixes
- Weiner (1973) and McCreight (1976) independently invented the suffix tree
  - a tree formed by putting all suffixes of T together



Suffix Tree of acacaac#

Definition of a Suffix Tree

- Suffix tree is an edge-labeled compact tree (no degree-1 nodes) with n leaves
  - each leaf ⇔ suffix
  - leaf label ⇔ starting pos of suffix
  - If we traverse from root to leaf k :
    edge labels along path ⇔ suffix T[k..n]
  - edge-label to each child starts with different character

Searching in a Suffix Tree

Theorem: If a pattern P occurs at position j in T, P is a prefix of T[j..n]

This suggests the searching algorithm below:

- Start from root of the suffix tree
- Traverse the suffix tree using P

➔ What we are doing is to match P with all suffixes of T at the same time

Searching in a Suffix Tree

Theorem: Pattern P occurs in T if and only if all chars of P are matched in the traversal of the searching algorithm

Questions:

1. How to locate the occurrences?
2. What is the searching time?

   $O(|P|+r)$ time, where r = #occurrences

Space Usage

- There are O(n) nodes and O(n) edges in the suffix tree
  → O(n) space ?
- Each edge needs to store its label, which can contain O(n) chars
  → In the worst-case, total O(n^2) chars
- Can we reduce space usage?

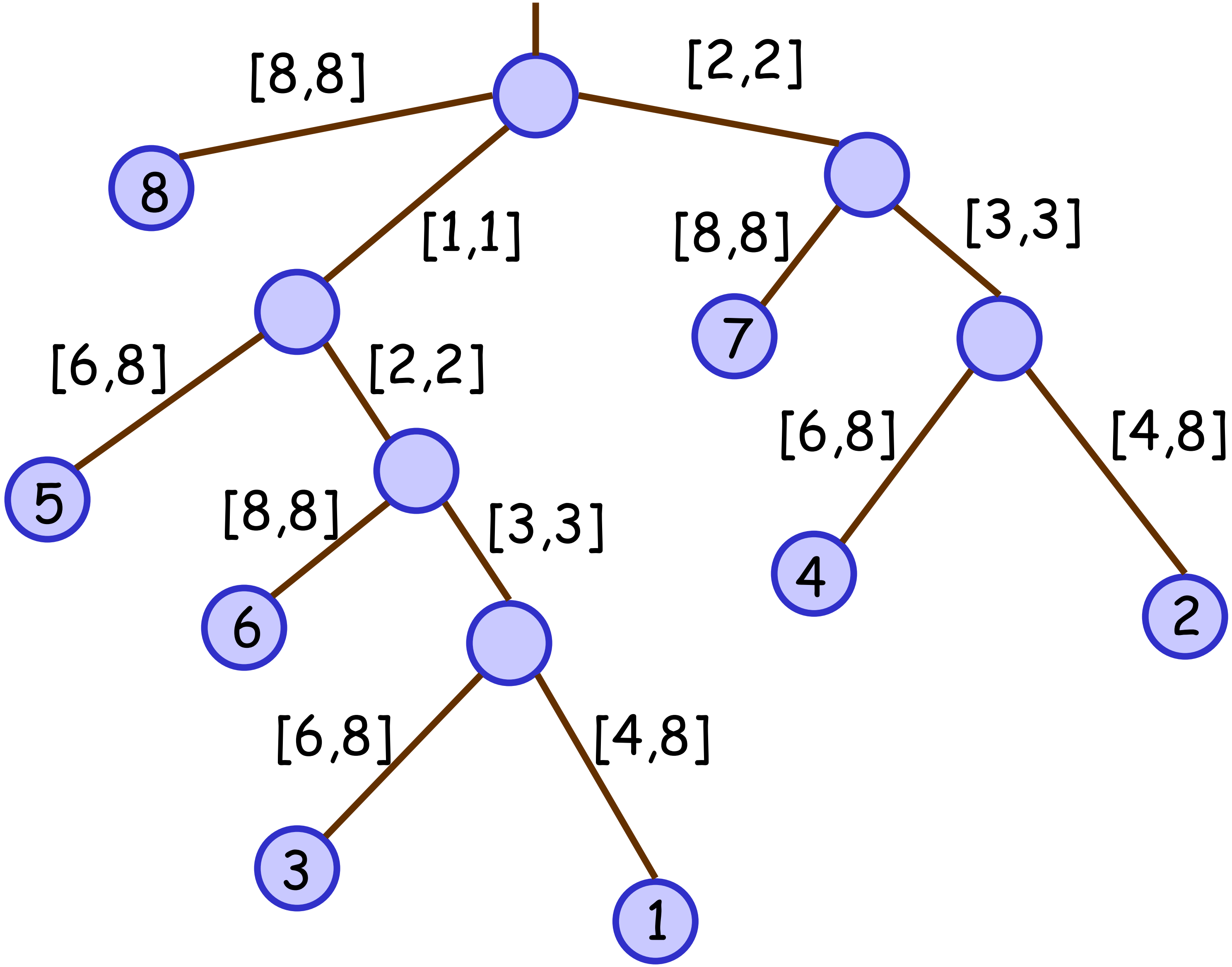
Space Usage

Observation: Each edge label must be equal to some substring of T

Clever Idea:

1. Store T, and
2. Replace each edge label by 2 integers, telling which substring it is equal to

→ Total space: $O(n)$



Suffix Tree of acacaac#

Suffix Array

- Although suffix tree takes O(n) space, the hidden constant is quite large
  ➔ around 40n to 60n bytes

- Manber and Myers (1990) simplified the suffix tree, and invented the suffix array
  - An array storing the suffixes of T in the "dictionary" order

Suffix Array

Suffix Array of acacaac#

1	#
2	aac#
3	ac#
4	acaac#
5	acacaac#
6	c#
7	caac#
8	cacaac#

- The suffix array SA for T has n entries
- For any j, SA[j] stores the j^th^ smallest suffix, based on alphabetical order
- Theorem: If P occurs in T, its occurrences correspond to consecutive region in SA

Suffix Array

Suffix Array of acacaac#

1	#
2	aac#
3	ac#
4	acaac#
5	acacaac#
6	c#
7	caac#
8	cacaac#

➔ Searching P takes $O(|P| \log n)$ time using binary search

Space:

We can represent each suffix by its starting position ➔ $O(n)$ space

In practice, around 14n bytes