Fast Resource Allocation for Downlink NOMA Based on Revenue and Chordal Graphs

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***Abstract*—Non-orthogonal multiple access (NOMA) is a promising technology for future mobile networks due to superior spectrum efficiency. In this paper, we model the interaction between the base station and multiple users as a Stackelberg game and devise a fast resource allocation method consisting of resource block allocation and power allocation. The objective is to serve as many users as possible at rates beyond their requirements, while seeking to enhance total revenue. We derive a closed-form formula for optimal power allocation. Besides, we convert the resource block allocation problem into the problem of finding a maximum weight independent set for a chordal graph, which only takes linear time. Simulation results show that the fast method we propose outperforms existing algorithms in total throughput and the number of users whose rate demands are attained.**

***Keywords*—non-orthogonal multiple access (NOMA), power allocation, resource allocation**

I. Introduction

With the increasingly rapid proliferation of mobile devices, it is expected that by 2021 the monthly global mobile data traffic will reach 49 exabytes and annual traffic will exceed half a zettabyte [1]. The high demand in mobile data traffic will worsen scarcity of spectrum resource. To overcome such difficulty, there have been a number of technologies proposed for future wireless communications development. Important technologies include device-to-device communications [2], massive multiple-input multiple output [3], NOMA [4], etc.

NOMA has been considered as a promising technology for the fifth generation (5G) of mobile networks because of its superior spectral efficiency, total throughput, and cell-edge throughput [6]. With NOMA technology, the base station multiplexes multiple users' signals over a sub-channel. Due to the nature of multiplexing, sub-channel (or resource unit/block) assignment and power allocation play key roles in overall performance. Recent studies have investigated resource scheduling and power control for downlink NOMA system. The objectives are throughput maximization [7]-[8], energy efficient maximization [9], proportional fairness [10]-[11], etc.

[7]-[8] which aim to maximize total throughput divide the joint sub-channel assignment and power allocation problem into two sub-problems. In [7], the sub-channel assignment sub-problem is formulated as a many-to-many two-side matching problem and is solved by a suboptimal, Gale-Shapley-based algorithm, called USMA. The power allocation sub-problem in [7] is dealt with by using the water-filling algorithm. In [8], the power allocation sub-problem is solved by an iterative, two-phase, water-filling-based method; while the sub-channel assignment sub-problem is solved by a greedy algorithm which assigns each sub-channel an equal number of users. In [14], sub-channel assignment uses the binary dislocation principle (BDP). After sorting N users in ascending order of their channel condition, BDP pairs the first user with the $(N/2)^{\text{th}}$ user, the second user with the $(N/2+1)^{\text{th}}$ user, and so on.

Unlike [7]-[8] which aim at throughput maximization, [9] aims to maximize energy efficiency for downlink NOMA. The authors consider the sub-channel allocation as a two-side matching process between the set of users and the set of sub-channels and propose a low-complexity suboptimal matching scheme called SOMSA. Similar to USMA in [7], SOMSA is a based on the Gale-Shapley algorithm. For energy-efficient power allocation, the authors in [9] convert the problem to the difference of convex (DC) representation and iteratively get a suboptimal solution by DC programming. Power allocation among sub-channels in is also solved by DC programming.

[10] focuses on user pairing and power allocation for 2-user NOMA. The authors consider all possible user pairs and then decide user scheduling and power allocation based on proportional fairness (PF). In [11], the authors proposed a water-filling based proportional fairness scheduling scheme to maximize the average throughput. This scheme consists of a number of stages; each stage allocates one more sub-channel than the previous stage does. In each stage, brute-force search helps to choose the user pair with the highest proportional fairness metric. Then transmission power is re-distributed over allocated sub-channels by water-filling. [15] also considers proportional fairness; users are scheduled by a meta-heuristic search algorithm which runs iteratively until convergence.

Unlike the aforementioned papers whose objectives are to optimize total throughput, energy efficiency, or proportional fairness for NOMA systems, the goal of this paper is to maximize the number of user pairs that attains their rate demands concurrently, while seeking to enhance total revenue. To this end, we consider resource block (RB) allocation and power allocation jointly. For RB allocation, we convert the RB allocation problem to the problem of finding out a maximum weight independent set (MWIS) for a chordal graph. Although the MWIS problem in general is NP-hard, obtaining a MWIS for chordal graphs takes linear time only. Hence our RB allocation scheme is fast. For power allocation, we model the interaction between the base station and users as a Stackelberg game and modify what is done in [5] with one major difference: Unequal weights are added to users' revenue, in order to encourage users with good channel condition to pair with users that have poor channel gain. Our power allocation scheme is

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very fast because we derive the closed-form formula for optimal transmission power.

The remainder of this paper is organized as follows. The system model is described in Section II. Section III introduces the method we propose, which consists of the revenue-based power allocation scheme and the chordal-graph-based RB allocation scheme. Performance evaluation is presented in Section IV. Section V gives some concluding remarks.

II. System Model

As shown in Fig. 1, we consider a downlink single-cell NOMA network,. where there are a base station (*BS*) and multiple users. The set of users is denoted by $\mathcal{M} = \{1, 2, \ldots, M\}$. Each user, say user m, can have its own rate demand, which is denoted by $R_m^{\min}$.

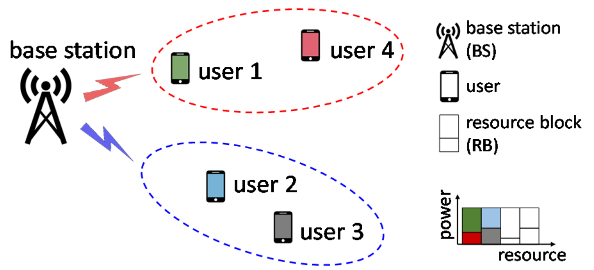

Fig. 1. Illustration of a single cell of NOMA networks. In this figure, there are two user pairs (drawn by circles)—one pair consists of users 1 and 4 and the other pair consists of users 2 and 3. The users in a pair use the same resource block by NOMA technology for their downlink transmissions.

The base station and users are equipped with a single antenna. The entire radio resource of the BS is divided into K resource blocks (*RB*s). For simplicity of exposition, the radio resource of the BS is assumed to be equally divided and thus each RB has the same transmission power, bandwidth, and/or time duration. The transmission power per RB is denoted by P_{RB}. The channel gain from the base station to the m-th user in the k-th RB is denoted by $h_{m,k}$ and its square is denoted by $G_{m,k} = |h_{m,k}|^2$. RBs can be heterogeneous; that is, $G_{m,i} \neq G_{m,j}$ is possible for $i \neq j$. However, when there is no ambiguity, G_m is also used to denote the squared channel gain for simplicity of exposition.

The BS serves users and attains their rate demands by *two-user NOMA* technology: In any single RB, two users can be served concurrently by superposition coding and successive interference cancellation (SIC), after resource block allocation (abbreviated as RB allocation) and power allocation have been determined. The RB allocation part allocates each RB to two users; while each user is assigned (at most) one RB. The power allocation part is on a per-RB basis. Given a RB and two users that are assigned the RB, the power allocation part determines the transmission power values to the two users.

In this paper, we propose a RB allocation scheme and a power allocation scheme. The primary goal is to maximize the number of users that concurrently attains their rate demands. If there are multiple optimal solutions for the primary goal, among these solutions we choose the one with highest revenue, which is the secondary goal of our schemes.

The following explains the encoding and decoding processes of two-user NOMA. Although the processes are on a per-RB, we do not explicitly point out which RB is under consideration for simplicity of exposition. Without loss of generality, let us assume that a *triple* is given which consists of user 1, user 2, and a certain RB. More precisely, it is known that a certain RB is allocated to both user 1 and user 2. We assume $G_1 \geq G_2$; therefore, user 1 is the *strong user* and user 2 is the *weak user*. Using superposition coding at the BS, the superposed signal received by user m, $m \in \{1, 2\}$, is

$$y_m = h_m \sum_{i=1}^{2} \sqrt{p_i} x_i + n_m \tag{1}$$

where p_i is the transmission power to user i from the BS, x_i is the signal the BS sends to user i, $n_m \sim CN(0, N_m)$ is the additive white Gaussian noise at user m, and N_m is the average noise power at user m.

SIC technology is applied to the decoding process. For the weaker user (user 2), the signal-to-interference-plus-noise ratio (SINR) is $\frac{G_2 p_2}{N_2 + G_2 p_1}$. So the achievable rate of the weak user is

$$R_2 = \log_2\left(1 + \frac{G_2 p_2}{N_2 + G_2 p_1}\right) \tag{2}$$

The strong user decodes its own signal after the weak user's signal has been decoded. Because the weak user's signal has been decoded and removed from the superposed signal received by the strong user, the achievable rate of the strong user is

$$R_1 = \log_2\left(1 + \frac{G_1 p_1}{N_1}\right) \tag{3}$$

In our power allocation scheme, the BS acts as the leader in the Stackelberg game (on a per-RB basis); the users act as the followers. (This is same as [5] except that we add unequal weights to users' revenue, which will be mentioned later.) The BS charges the strong user (user 1) a fee at a price of λ_1 per transmission power and the weak user (user 2) at a price of λ_2, in order to maximize its own revenue. In consequence, the BS earns a revenue of $\lambda_1 p_1$ from user 1 and a revenue of $\lambda_2 p_2$ from user 2. So the revenue maximization problem of the BS can be formulated as:

$$\begin{aligned} \max_{\lambda_1, \lambda_2} \ & U_{\text{BS}}(\lambda_1, \lambda_2) = \lambda_1 p_1 + \lambda_2 p_2 \\ \text{subject to } \ & p_1 + p_2 \leq P_{\text{RB}} \\ & p_i \geq 0,\ i = 1,2 \end{aligned} \tag{4}$$

Based on the price, strong user and weak user (users 1 and 2) determine their own transmission power values in order to maximize their own revenue. The revenue of user m is defined as $w_m R_m - \lambda_m p_m$, where R_m is the achievable rate shown in (2)-(3) and w_m is a weight taking a value in $(0, 1]$. The first term of user's revenue, $w_m R_m$, is the income earned from its achievable rate. The second term, $-\lambda_m p_m$, is the payment charged by the BS. The revenue maximization problem of user m, $m \in \{1, 2\}$, can be formulated as:

$$\begin{aligned} \max_{p_m} \ & U_m(p_m) = w_m R_m - \lambda_m p_m \\ \text{subject to } \ & 0 \leq p_m \leq P_{\text{RB}} \\ & R_m \geq R_m^{\min} \end{aligned} \tag{5}$$

III. Resource Allocation

The fast resource allocation method we propose consists of two parts—revenue-based power allocation and chordal-graph-based RB allocation, which are explained in the following.

A. Revenue-Based Power Allocation

The revenue-based power allocation scheme we develop is a modification of the price-based power allocation scheme in [5] with one major difference: Unequal weights are added to users' revenue, in order to encourage strong users to pair with weak users that have poor channel gain. By contrast, the price-based scheme proposed in [5], which adopts equal weights due to user pairing (and RB allocation) not considered, makes strong users tend to pair with the weak users with good channel condition. In the following, we will omit some details and only outline our power allocation scheme with emphasis on the difference from [5] and the results.

Suppose that we are given a triple consisting of a strong user, a weak user, and a RB that is allocated to these two users. Without loss of generality, we assume that the two users are user 1 and user 2. We also assume that $G_1 \geq G_2$; therefore, user 1 is the strong user and user 2 is the weak user. In our revenue-based power allocation scheme, the weight for user 1 (and any strong user) is set to $w_1 = \alpha$, where $\alpha = e^{-N_1 G_2/(N_2 G_1)} = e^{-\sigma_2/\sigma_1}$ and $\sigma_m = N_m/G_m$, $m \in \mathcal{M}$. The weight for user 2 (and any weak user) is set to $w_2 = 1$. So the revenue maximization problems for users 1 and 2 are

$$\begin{aligned} &\max_{p_1}\ U_1(p_1) = \alpha \log_2\left(1+\tfrac{G_1 p_1}{N_1}\right) - \lambda_1 p_1 \\ &\text{subject to}\ \ 0 \leq p_1 \leq P_{\mathrm{RB}} - p_2 \\ &\qquad\qquad \log_2\left(1+\tfrac{G_1 p_1}{N_1}\right) \geq R_1^{\min} \end{aligned} \tag{6}$$

and

$$\begin{aligned} &\max_{p_2}\ U_2(p_2) = \log_2\left(1+\tfrac{G_2 p_2}{N_2+G_2 p_1}\right) - \lambda_2 p_2 \\ &\text{subject to}\ \ 0 \leq p_2 \leq P_{\mathrm{RB}} - p_1 \\ &\qquad\qquad \log_2\left(1+\tfrac{G_2 p_2}{N_2+G_2 p_1}\right) \geq R_2^{\min} \end{aligned} \tag{7}$$

respectively.

We model the BS and the two users as the leader and followers in the Stackelberg game. Both users decide their best transmission power, based on the prices offered by the BS, so as to maximize their revenue. The BS which knows how the users decide their best transmission power can decide the prices accordingly so as to maximize the BS's revenue.

In the case without rate constraints, the two users can maximize their revenue by taking partial derivative. The optimal transmission values of p_1 and p_2 satisfy:

$$\lambda_1 = \alpha \cdot \frac{(\ln 2)^{-1} G_1}{N_1 + G_1 p_1} = \frac{1}{\ln 2} \cdot \frac{\alpha}{\sigma_1 + p_1} \tag{8}$$

$$\lambda_2 = \frac{(\ln 2)^{-1} G_2}{N_2 + G_2(p_1+p_2)} = \frac{1}{\ln 2} \cdot \frac{1}{\sigma_2 + p_1 + p_2} \tag{9}$$

Theorem 1. If $0 \leq p_1 \leq P_{\mathrm{RB}}$, $0 \leq p_2 \leq P_{RB} - p_1$, and $\sigma_2 > 0$,

$$\max_{p_2} \frac{p_2}{\sigma_2 + p_1 + p_2} = \frac{P_{\mathrm{RB}} - p_1}{\sigma_2 + P_{\mathrm{RB}}}$$

After substituting (8) and (9) into (4) and using Theorem 1, maximizing BS's revenue is equivalent to:

$$\max_{p_1}\ \widetilde{U}_{\mathrm{BS}}(p_1) = \frac{\alpha \cdot p_1}{\sigma_1 + p_1} + \frac{P_{\mathrm{RB}} - p_1}{\sigma_2 + P_{\mathrm{RB}}} \tag{10}$$

where $\widetilde{U}_{\mathrm{BS}}$ differs from U_{BS} by a factor of $\ln 2$. The derivative of $\widetilde{U}_{\mathrm{BS}}(p_1)$ is

$$\widetilde{U}'_{\mathrm{BS}}(p_1) = \frac{\alpha \cdot \sigma_1}{(\sigma_1 + p_1)^2} - \frac{1}{\sigma_2 + P_{RB}} \tag{11}$$

BS's revenue is maximized when $\widetilde{U}'_{\mathrm{BS}}(p_1) = 0$. Using (11) to solve $\widetilde{U}'_{\mathrm{BS}}(p_1) = 0$, we obtain user 1's optimal transmission power in the case without rate constraints is

$$\widehat{p_1} = -\sigma_1 + \sqrt{\alpha\sigma_1(\sigma_2 + P_{\mathrm{RB}})} \tag{12}$$

Now let us put the rate constraints back. The transmission power of user 1 must be large enough to satisfy its own rate demand; meanwhile, it has to be small enough to meet user 2's rate demand. The lower bound (denoted by p_1^l) and upper bound (denoted by p_1^u) for user 1's transmission power are:

$$\begin{aligned} p_1^l &= \sigma_1\left(2^{R_1^{\min}} - 1\right) \\ p_1^u &= \min\left\{\frac{\sigma_2 + P_{RB}}{2^{R_2^{\min}}} - \sigma_2, P_{RB}\right\} \end{aligned} \tag{13}$$

p_1^*, which is defined as the optimal transmission power of user 1 with rate constraints taken into account, can be calculated by:

$$p_1^* = \begin{cases} \widehat{p_1} & ,\text{if } p_1^l \leq \widehat{p_1} \leq p_1^u \\ p_1^l & ,\text{if } \widehat{p_1} < p_1^l \\ p_1^u & ,\text{if } \widehat{p_1} > p_1^u \end{cases} \tag{14}$$

$p_1^l \leq p_1^u$ is the necessary and sufficient condition of existing feasible solutions under rate and power constraints. Combining (10) and (14), the maximum revenue of the BS can be computed:

$$U_{\mathrm{BS}}^* = \begin{cases} \frac{1}{\ln 2} \widetilde{U}_{\mathrm{BS}}(p_1^*) & ,\text{if } p_1^l \leq p_1^u \\ 0 & ,\text{otherwise} \end{cases} \tag{15}$$

B. Chordal-Graph-Based RB Allocation

Our RB allocation scheme attempts to assign two users to each RB, with the help of a conflict graph $G = (V, E)$. As illustrated in Fig. 2, a vertex in the conflict graph is a triple consisting of a strong user, a weak user, and a RB that is allocated to these two users. For example, the leftmost vertex in Fig. 2 denotes that RB 5 is allocated to both users 6 and 10 and denotes that user 6 is the strong user and user 10 is the weak user. In the conflict graph, an edge connecting two vertices means that the two vertices have one or more elements in common, which can be either one or two users in common, a RB in common, or a combination thereof.

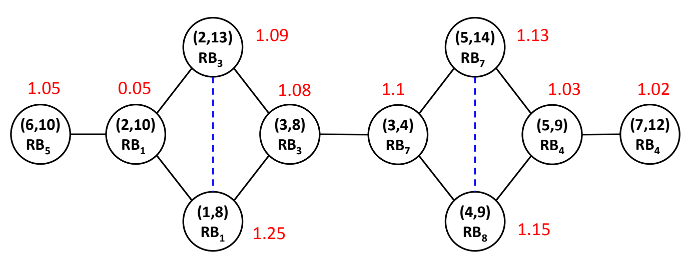

Fig. 2. The vertices and solid edges comprise a conflict graph. After adding the dashed edges, the conflict graph becomes a chordal graph. Vertex weights are denoted by (red) numbers around vertices.

Each vertex in the conflict graph corresponds a *well-served triple*; a triple not well-served does not appear in the conflict graph. A triple is well-served if the two users in the triple can both be served in the RB at rates no lower than their own rate demands. According to Section III.A, a triple is well-served if and only if $p_1^l \le p_1^u$ (or equivalently $U_{BS}^* > 0$) for the triple, where p_1^l and p_1^u are computed by (13) and U_{BS}^* by (15).

The primary goal of our RB allocation scheme is equivalent to maximizing the number of *concurrently well-served triples*. Concurrently well-served triples are well-served triples that do not have any element in common. In the conflict graph, the set of concurrently served triples is an independent set, because there is no edge connecting them. If there are multiple optimal solutions for the primary goal, we choose the one with highest revenue, which is the secondary goal. Finding out an optimal solution for these goals is equivalent to solving a maximum weight independent set (*MWIS*) problem, where the weight W_v of vertex v is set to:

$$W_v = 1 + \frac{\text{BS's max revenue for vertex } v}{\sum_{i \in V} \text{BS's max revenue for vertex } i} \quad (16)$$

The first term of the right-hand side in (16) corresponds to the basic utility of a well-served triple. The second term is the extra utility of a well-served triple brought from the triple's revenue. which is calculated by (15).

The MWIS problem is NP-hard; however, its complexity becomes lower for chordal graphs. A chordal graph is a graph in which every induced cycle has exactly three vertices. Finding a MWIS for chordal graphs takes linear time, thus motivating us to devise a RB allocation algorithm based on chordal graphs.

As shown in Fig. 3, our RB allocation algorithm runs iteratively. In the initialization, V, which is the set of well-served triples that still wait for processing, is set to be the set of all well-served triples. S, the set of concurrently well-served triples we have already found, is initialized to be an empty set.

Each iteration starts with constructing a conflict graph, denoted by $G = (V, E)$. In order to obtain a chordal graph G_c, we test the chordality and add edges to G if G is not chordal. This is done by executing the maximum cardinality search algorithm in [12] and the fill-in computation algorithm in [12] sequentially. After that, we feed the chordal graph G_c into Algorithm 1 in [13] (which takes linear time) to obtain a MWIS, denoted by I. I corresponds to the concurrently well-served triples we find out at this iteration; hence, we add I to S.

I is a MWIS of G_c, not necessarily a MWIS of the original conflict graph. It is possible that some well-served triple not in S can be well-served concurrently with the triples in S; going to the next iteration helps to find out such well-served triple(s). Because the vertices in I and their neighboring vertices cannot be well-served concurrently with the triples in S, all these vertices are removed from V. If V is not empty, our algorithm repeats by going to the next iteration; otherwise, our algorithm completes. Upon completion, S is the output of our RB allocation algorithm, which gives the set of concurrently well-served triples.

Algorithm: Our chordal-graph-based RB allocation

```
// Initialization.
S ← ∅  // S is the set of concurrently well-served triples found.
V ← ∅  // V is the set of well-served triples waiting for processing.
for each possible triple v      // V is initialized to be the set of all
                                   well-served triples.
   Calculate p_1^l and p_1^u by (13).
   if p_1^l ≤ p_1^u then
      V ← V ∪ {v}
   end if
end for
// Iterations start here.
while V ≠ ∅
   For V, construct a conflict graph G = (V, E).
   Starting from G, add edges to get a chordal graph G_c by running
      maximum cardinality search and fill-in computation in [12].
   Get the MWIS (denoted by I) by using Algorithm 1 in [13].
   S ← S ∪ I
   V ← V \ I \ N(I)        // Remove all vertices in I and their
                              neighbors from V.
end while
```

Fig. 3. Pseudo code of our chordal-graph-based RB allocation scheme.

IV. Performance Evaluation

By simulation, we compare the performance of our method with the performance of several existing algorithms including the channel state sorting-pairing algorithm (*CSS-PA*) [14] and the population-based meta-heuristic search algorithm (*meta-heuristic*) [15]. All these algorithms are designed for NOMA.

Important performance metrics include the number of (concurrently) well-served users, total throughput, and run time. For fairness, we set $G_{m,i} = G_{m,j}$ and $N_i = N_j$, $\forall m, i, j$, and evaluate total throughput instead of total revenue. This is because among these algorithms, our method is the only one that considers revenue maximization and heterogeneous RBs.

The simulation is set as follows. We consider a single cell with a radius of 500 meters. In the cell, users are randomly deployed with uniform distribution. The rate requirement of user m, $m \in \mathcal{M}$, range from 1 to 8 bps/Hz:

$$R_m^{\min} = \begin{cases} 8 & \text{, if } \log_2(1 + G_m P_{RB}/N_m) \ge 16 \\ 4 & \text{, if } 8 \le \log_2\left(1 + \frac{G_m P_{RB}}{N_m}\right) < 16 \\ 2 & \text{, if } 4 \le \log_2\left(1 + \frac{G_m P_{RB}}{N_m}\right) < 8 \\ 1 & \text{, if } \log_2\left(1 + \frac{G_m P_{RB}}{N_m}\right) < 4 \end{cases}$$

The maximum transmission power of BS is 40 dBm. Other important simulation parameters are listed in Table 1.

TABLE 1. Simulation Parameters

Radius of cell	500 m
Minimum rate requirement of users	1, 2, 4, 8 bps/Hz
Path loss model	$133.6 + 35\log_{10} d$[km]
The number of resource blocks	20
Bandwidth per resource block	180 KHz
Noise spectral density	-174 dBm/Hz

As shown in Fig. 4 and Fig. 5, our method performs significantly superiorly to the other algorithms in terms of both the number of (concurrently) well-served users and total throughput. Meta-heuristic is runner-up and CSS-PA performs worst. Besides, it is observed that the outperformance of our method becomes larger as the total number of users increases.

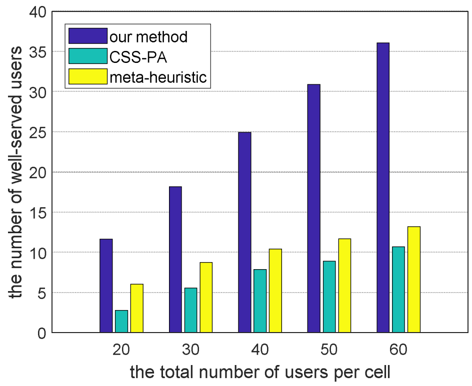

Fig. 4. The performance in terms of the number of well-served users.

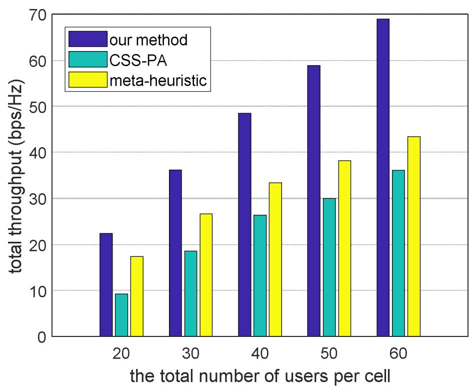

Fig. 5. The performance in terms of total throughput.

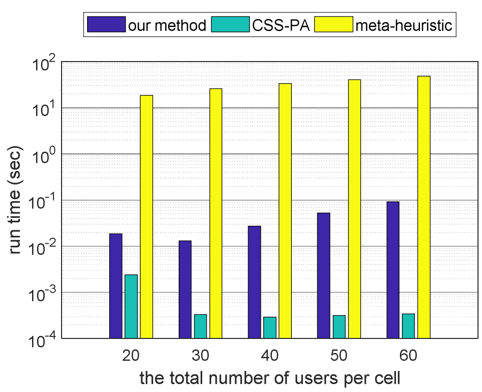

Fig. 6. Run time versus the number of users in a cell.

In terms of run time, it is observed in Fig. 6 that our method is much faster than the meta-heuristic algorithm. This is because that the meta-heuristic algorithm takes a long time to converge. Our method is slower than the CSS-PA algorithm because as mentioned in Section I, CSS-PA relies on binary dislocation user pairing, which is very simple. Although CSS-PA is fastest among the algorithms, its performance is the worst in terms of both total throughput and the number of well-served users.

V. CONCLUSION

This paper studies resource allocation for downlink NOMA networks and proposes a fast method which consists of power allocation and RB allocation. The primary goal is to maximize the number of users whose rate demands are met and the secondary goal is to maximize total revenue. To achieve these goals, we model the interaction between the base station and users as a Stackelberg game and derive a closed-form formula for optimal transmission power. Besides, we devise an iterative, chordal-graph-based RB allocation scheme in which each iteration takes linear time. Simulation results show that the fast method we propose outperforms several algorithms in terms of total throughput and the number of users whose rate requirements are attained.

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