Ratio Rules: A New Paradigm for Fast, Quantifiable Data Mining

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Proceedings of VLDB Conference, 1998.

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Mining Quantitative Association Rules in Large Relational Tables

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Proceedings of ACM SIGMOD Conference, 1996.

Outline

- **Introduction**
- **Ratio Rule Discovery**
- **Prediction of Missing Values**
- **Measurement of the Goodness**
- **Experiments**
- **Discussion**

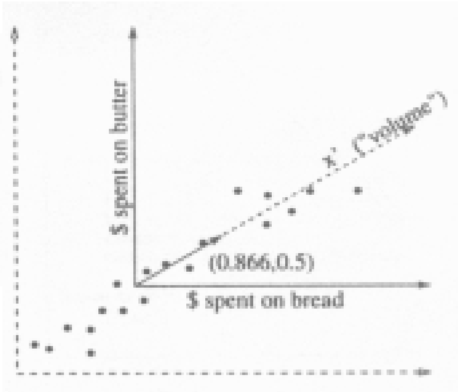
Introduction

- **Paradigms**
  - boolean association rule
    - **{bread,milk} ⇒ butter (90%)**
  - quantitative association rule
    - **bread:[3-5] and milk:[1-2] ⇒ butter:[1.5-2]**
  - ratio rule
    - **bread:milk:butter=1:2:5**
    - **applications**
      - *data cleaning, forecasting, decision support, outlier detection, visualization*

Introduction

- **An example**

customer	*bread* ($)	*butter* ($)
Billie	.89	.49
Charlie	3.34	1.85
Ella	5.00	3.09
...	...	...
John	1.78	.99
Miles	4.02	2.61



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Introduction

- **Innovations**
  - ■ a single-pass algorithm for ratio rule discovery
    - ◆**eigensystem analysis**
  - ■ a method to predict missing/hidden values from the ratio rules
    - ◆**linear algebra**
  - ■ a measure of the goodness for a set of rules
    - ◆**guessing error**

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Introduction

- **Notations**

symbol	definition
N	number of records
M	number of attributes
k	cutoff (number of Ratio Rules retained)
h	number of holes
$\mathcal{H}$	set of cells which have holes
$\mathcal{R}$	set of rules
GE_1	guessing error over each hole
GE_h	guessing error over h holes
$\times$	matrix multiplication
$\mathbf{X}$	the $N \times M$ data matrix
$\mathbf{X}_c$	the centered version of $\mathbf{X}$
$\mathbf{X}^t$	the transpose of $\mathbf{X}$
$x_{i,j}$	value at row i, column j of the matrix $\mathbf{X}$
$\hat{x}_{i,j}$	reconstructed (approximate) value at row i and column j
$\bar{x}$	the mean cell value of $\mathbf{X}$
$\mathbf{C}$	the $M \times M$ covariance matrix ($\mathbf{X}_c^t \times \mathbf{X}_c$)
$\mathbf{V}$	the $M \times k$ RR matrix

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Ratio Rule Discovery

- **Eigensystem analysis**
  - compute the eigenvalues and eigenvectors of the covariance matrix for the given data points
  - identify the axes of greatest variance
  - reduce the dimensionality of a data set while retaining as much as variation as possible
    - ◆**only the eigenvectors of the k largest eigenvalues are used as the ratio rules**

$$\frac{\sum_{i=1}^{k} \lambda_i}{\sum_{j=1}^{M} \lambda_j} \approx 85\%$$

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Ratio Rule Discovery

- **Proposed Method**
  - zero-mean the input matrix to derive X_c
  - compute C $C \equiv X_c^t \times X_c$
  - compute the eigenvalues and eigenvectors of C
- **An example**
  - N=4, M=2, k=1 (column averages:[2 3])

$$X = \begin{bmatrix} 2 & 4 \\ 3 & 3 \\ 3 & 4 \\ 0 & 1 \end{bmatrix} \Rightarrow X_c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \\ -2 & -2 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 6 & 5 \\ 5 & 6 \end{bmatrix}$$

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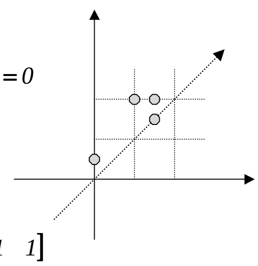
Ratio Rule Discovery

- **Results**
  - the largest eigenvalues: λ_1=11
  - the first ratio rule (RR): [1 1]

$$det\left(\begin{bmatrix} 6-\lambda & 5 \\ 5 & 6-\lambda \end{bmatrix}\right) = 0 \Rightarrow (6-\lambda)(6-\lambda) - 25 = 0$$

$$\Rightarrow \lambda^2 - 12\lambda + 11 = 0 \Rightarrow \lambda = 11 \vee 1$$

$$\begin{bmatrix} 6-11 & 5 \\ 5 & 6-11 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0 \Rightarrow \nu_1 = [y_1 \quad y_2] = [1 \quad 1]$$



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Ratio Rule Discovery

- **Single-pass algorithm**

```
/* input: training set X on disk */
/* output: covariance matrix C */
for j := 1 to M do
    colavgs[j] ← 0;
    for l := 1 to M do
        C[j][l] ← 0;
for i := 1 to N do
    Read ith row of X from disk (X[i][1],...,X[i][M]);
    for j := 1 to M do
        colavgs[j] += X[i][j];
        for l := 1 to M do
            C[j][l] += X[i][j]*X[i][l];
for j := 1 to M do
    colavgs[j] /= N;
for j := 1 to M do
    for l := 1 to M do
        C[j][l] -= N * colavgs[j] * colavgs[l];
```

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Ratio Rule Discovery

- **Eigensystem computation**

```
input:
    covariance matrix C in main memory

output:
    eigenvectors v_1, ..., v_k (i.e., the RRs)

compute eigensystem:
    {v_1, ..., v_M} ← eigenvectors(C);
    {λ_1, ..., λ_M} ← eigenvalues(C);
    sort v_j according to the eigenvalues;
    choose k based on Eq. 1;
    return the k largest eigenvectors;

complexity:
    O(M^3)
```

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Prediction of Missing Values

- **Definitions**
  - h-hole row vector b_H
    - ◆**$b_{\{2,4\}}$=[x_1 ? x_3 ? x_5] (h=2)**
  - (M-h)×M elimination matrix E_H
    - ◆**$E_{\{2,4\}}$=** $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$
- **Basic idea**
  - fill the unknowns by the ratio rules in $E_H \times V$ and the partial knowledge in $E_H \times b_H^t$

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Prediction of Missing Values

- **Pseudo code**

```
/* input:  b_H, a 1 × M row vector with holes */
/* output: b̂, a 1 × M row vector with holes filled */
1.  V' ← E_H × V;                          /* ''RR-hyperplane'' */
2.  b' ← E_H × b_H^t;                      /* ''feasible sol'n space'' */
3.  solve V' × x_concept = b' for x_concept  /* solution in k-space */
4.  d ← V × x_concept;                      /* solution in M-space */
5.  b̂ ← b × [E_H^c]^t + d × [E_H]^t;
```

- **2-D example**
  - $b_{\{2\}}$=[1 ?], $E_{\{2\}}$=[1 0], V=[1 1]^t
  - V'=[1], b'=[1], $x_{concept}$=[1], d=[1 1]

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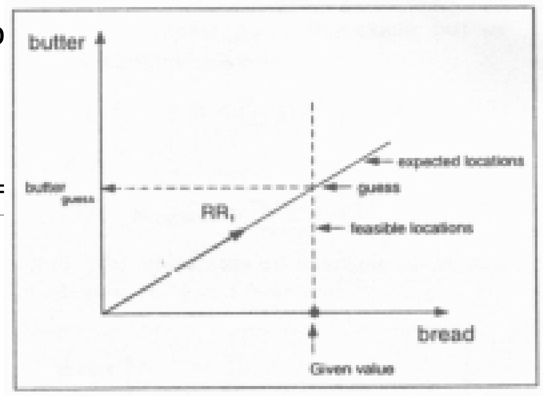
Prediction of Missing Values

- **Case 1**
  - exactly-sp
  - M-h=k
  - one exact
    - ◆**M=2, h=**



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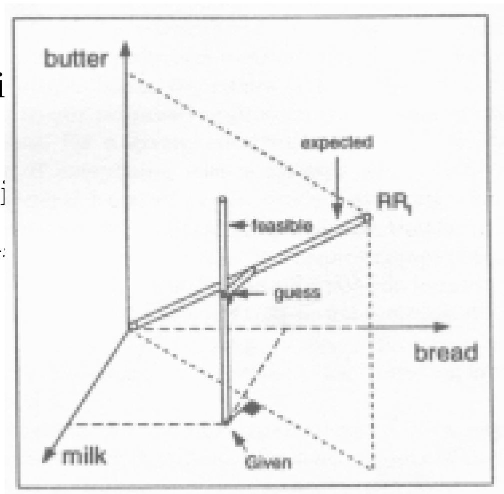
Prediction of Missing Values

- **Case 2**
  - over-specifi
  - M-h>k
  - no intersecti
    - ◆**M=3, h=1**



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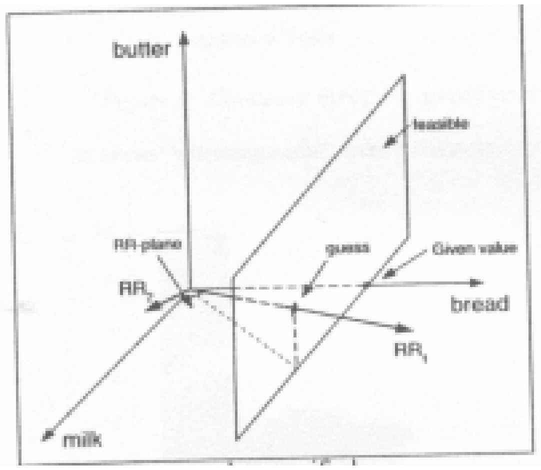
Prediction of Missing Values

- **Case 3**
  - under-s
  - M-h<k
  - infinite
    - ◆**M=3,**



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Measurement of the Goodness

- **Guessing error**

$$GE_1 = \sqrt{\frac{1}{NM}\sum_{i}^{N}\sum_{j}^{M}\left(\hat{x}_{ij} - x_{ij}\right)^2} \qquad GE_h = \sqrt{\frac{1}{Nh|H_h|}\sum_{i}^{N}\sum_{H \in H_h}\sum_{j \in H}\left(\hat{x}_{ij} - x_{ij}\right)^2}$$

- **Examples**

$$X = \begin{bmatrix} 2 & 4 \\ 3 & 3 \\ 3 & 4 \\ 0 & 1 \end{bmatrix} \Rightarrow \hat{X} = \begin{bmatrix} 4 & 2 \\ 3 & 3 \\ 4 & 3 \\ 1 & 0 \end{bmatrix} \Rightarrow \begin{aligned} &GE_1(X) \\ &= \sqrt{2^2 + 2^2 + 0 + 0 + 1^2 + 1^2 + 1^2 + 1^2} \\ &= 2\sqrt{3} \end{aligned}$$

$$X_1 = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \Rightarrow \hat{X}_1 = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} \Rightarrow \begin{aligned} &GE_1(X_1) = \sqrt{0 + 0 + 2^2 + 2^2} \\ &= 2\sqrt{2} \end{aligned}$$

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Experiments

- **Data sets**
  - NBA (452×12)
    - **minutes played, field goals, rebounds, fouls**
  - baseball (1574×17)
    - **batting average, at-bats, hits, home runs**
  - abalone (4177×7)
    - **length, diameter, weights**
- **Competitor**
  - column averages

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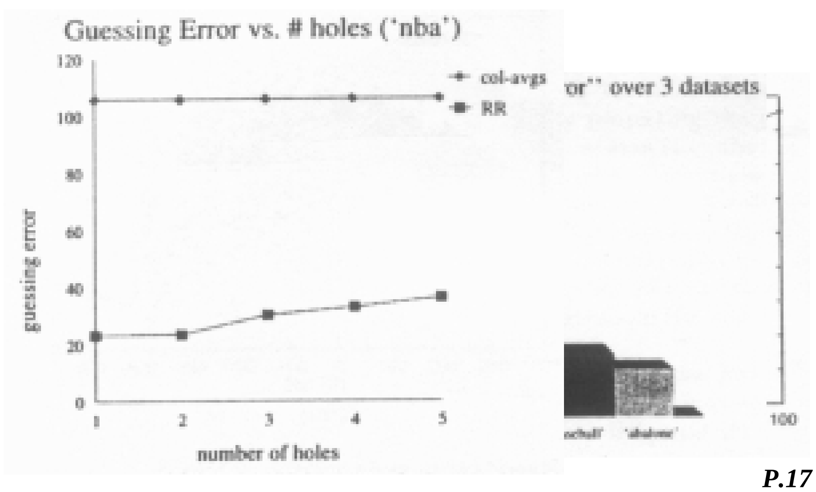
Experiments



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Discussion

- **Interpretation**
  - RR_1
    - **court action**
  - RR_2
    - **field position**
  - RR_3
    - **height**

field	RR_1	RR_2	RR_3
minutes played	.808	−.4	
field goals			
goal attempts			
free throws			
throws attempted			
blocked shots			
fouls			
points	.406	.199	
offensive rebounds			
total rebounds		−.489	.602
assists			−.486
steals			−.07

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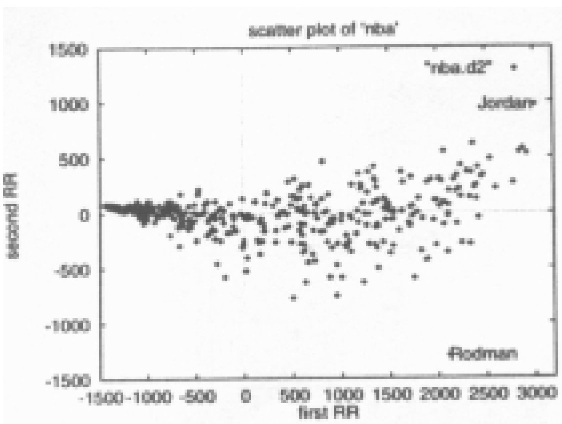
Discussion

- **Visualization**
  - RR_1
    - **Jordan**
  - RR_2
    - **Jordan**
    - **Rodman**



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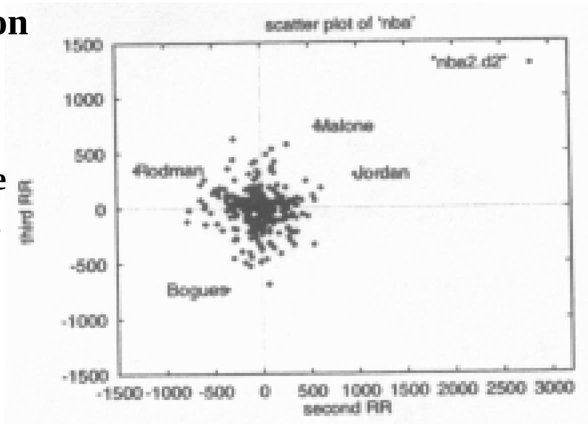
Discussion

- **Visualization**
  - RR_2
  - RR_3
    - ◆**Malone**
    - ◆**Bogues**



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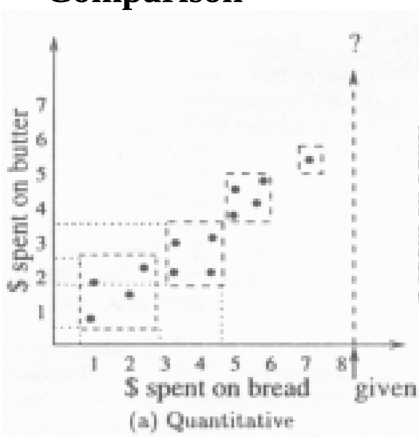
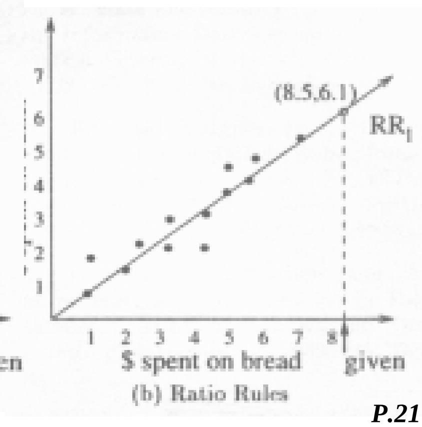
Discussion

- **Comparison**





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Discussion

- **Advantages of ratio rules**
  - achievement of more compact descriptions if the data points are linearly correlated
  - prediction of one or more unknown values when a new data record is given
  - measure of the guessing error, which can quantify how good a given set of rules is
  - easy to implement
  - only a single pass over the data set is required

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