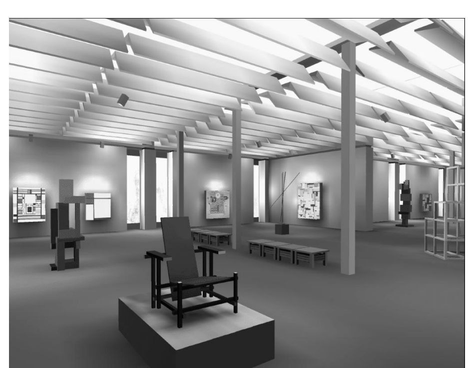
Radiosity

What is Radiosity

- Borrowed from radiative heat transfer.
- Assuming diffuse reflectance.
- View independent solution.



Rendering Equation (Review)

$$I(x,x') = \varepsilon(x,x') + \int_s \rho(x,x',x'')I(x',x'')dx''$$

- g() removed.
- ρ() is related to BRDF.

Radiosity Equation – Single Patch

$$B_i = E_i + R_i \int_j B_j F_{ij}$$

- Discrete form: $$B_i = E_i + R_i \sum_j B_j F_{ij}$$
- Compared to Rendering Equation:

$$I(x,x') = \varepsilon(x,x') + \int_s \rho(x,x',x'')I(x',x'')dx''$$

Radiosity Equation – In Matrix Form

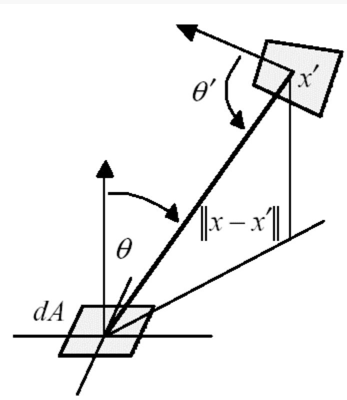
$$B_i = E_i + R_i \sum_j B_j F_{ij}$$

$$\Rightarrow \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} R_1F_{11} & R_1F_{12} & \cdots & R_1F_{1n} \\ R_2F_{21} & R_2F_{22} & \cdots & R_2F_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ R_nF_{n1} & R_nF_{n2} & \cdots & R_nF_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

$$\begin{bmatrix} 1-R_1F_{11} & -R_1F_{12} & \cdots & -R_1F_{1n} \\ & 1-R_2F_{22} & \cdots & -R_2F_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ -R_nF_{n1} & -R_nF_{n2} & \cdots & 1-R_nF_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

Form Factor Calculation

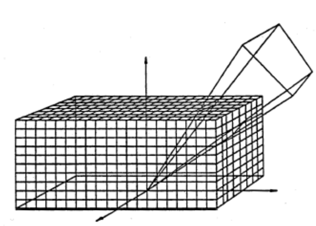
$$F_{ij} = \frac{1}{A_i} \int_{Ai} \int_{Aj} \frac{\cos\theta i \cos\theta j}{\pi |x - x'|^2} dA_j dA_i$$

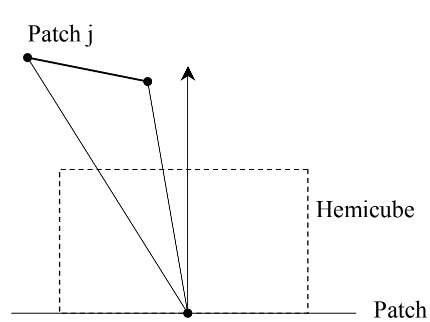


Form Factor Calculation

- Analytical forms of form factors are possible for simple shapes only.
- Form factor is reduced if a patch sees the other patch partially.
- An approximation – Hemicube.
- One hemicube for each patch.

Hemicube





Solving the Linear Equation

- Direct method: Gaussian Elimination $O(n^3)$
- Iterative method: Gauss-Seidel or Jacobi method $O(n^2)$

Jacobi Iterations

$$Ax = E$$

$$x_i^{(k+1)} = \frac{E_i - a_{i1}x_1^{(k)} - \ldots - a_{1n}x_n^{(k)}}{a_{ii}}$$

- Take a initial guess $\{x_i^0\}$, for i=1..n
- Each iteration produces better $\{x_i^{(k+1)}\}$ from $\{x_i^{(k)}\}$

Gauss-Seidel Variation

- New x_i's in iteration k+1 are used whenever they are available.
- That is: $x_i^{(k+1)}$ uses $x_1^{(k+1)} \ldots x_{i-1}^{(k+1)}$ and $x_{i+1}^{(k)} \ldots x_n^{(k)}$

Iteration k+1

Iteration k

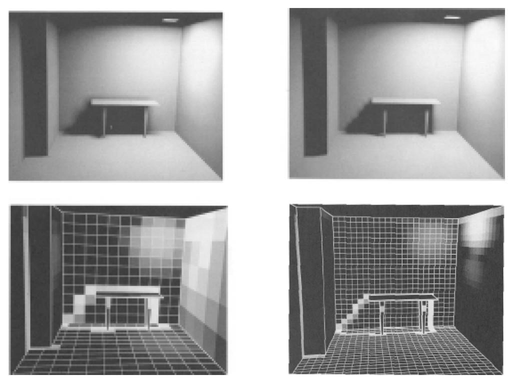
Progressive Refinement

- Allowing the viewing of early incomplete solution
- Start from the patch with greatest unshot radiosity.
- Update receiving patches. Repeat.

Artifacts

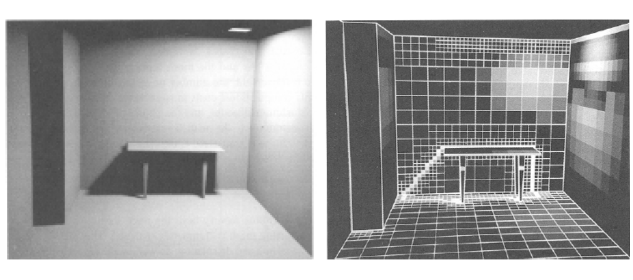
- Interpolation necessary to smooth out the patches.
- Blocky shadow is still a problem.
- Meshing resolution is important to the final quality.

Meshing



Adaptive Meshing

- Subdivide a patch if the radiosity variation is large.



Advanced Techniques

- Discontinuity Meshing.
- Hierarchical Radiosity.
- See Watt's Section 11.7.2 for details.

For Further Information

- Watt's book 10.3.2 and Chapter 11.
  (Pharr's book doesn't cover radiosity because it mainly uses Monte Carlo path tracing.)
- "Radiosity OverView Part 1" SIGGRAPH 1993 Education Slide Set, by Stephen Spencer. (Link)