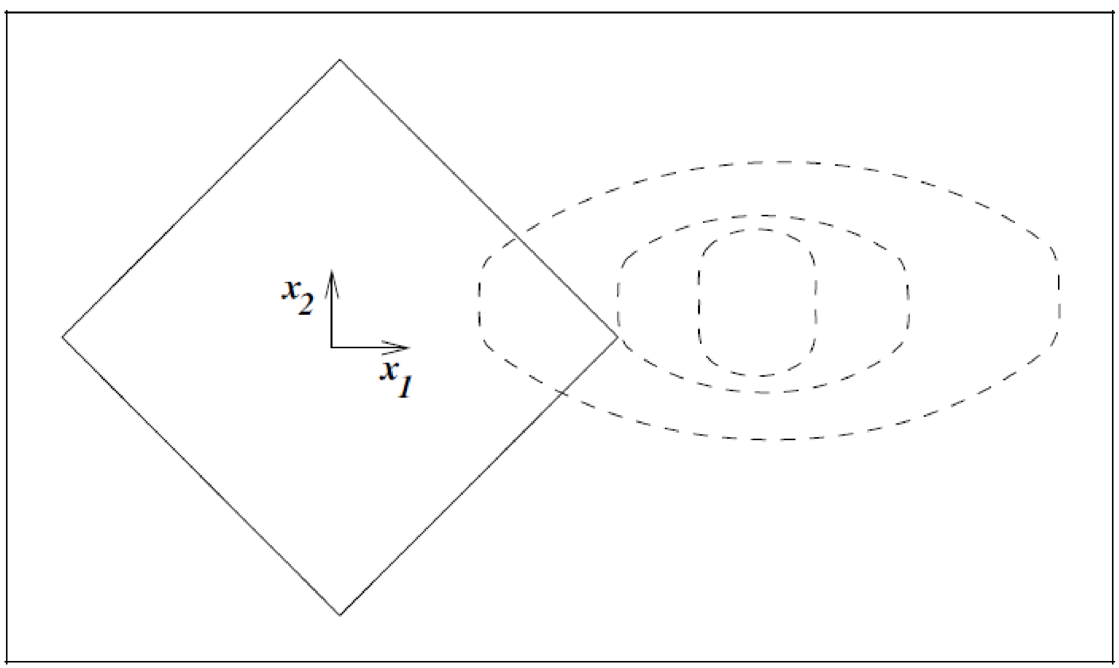
1. KKT condition (pg322)

$$\min_{x_1, x_2} \left(x_1 - \frac{3}{2}\right)^2 + \left(x_2 - \frac{1}{2}\right)^4 \quad \text{s.t.} \quad \begin{bmatrix} 1 - x_1 - x_2 \\ 1 - x_1 + x_2 \\ 1 + x_1 - x_2 \\ 1 + x_1 + x_2 \end{bmatrix} \geq 0.$$



- The first two constraints are active. Others are inactive

$$\nabla f(x^*) = \begin{bmatrix} -1 \\ -\frac{1}{2} \end{bmatrix}$$

$$\nabla c_1(x^*) = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\nabla c_2(x^*) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

- The Lagrangian multiplier
  - Check KKT conditions.

$$\lambda^* = \left(\tfrac{3}{4}, \tfrac{1}{4}, 0, 0\right)^T$$

2. Second order conditions

$$\min -0.1(x_1 - 4)^2 + x_2^2 \qquad 1 - x_1^2 - x_2^2 \geq 0$$

$$L(x, \lambda) = -0.1(x_1 - 4)^2 + x_2^2 - \lambda(1 - x_1^2 - x_2^2)$$

$$\nabla_x L(x, \lambda) = \begin{pmatrix} -0.2(x_1 - 4) + 2\lambda x_1 \\ 2x_2 + 2\lambda x_2 \end{pmatrix}$$

$$\nabla_{xx} L(x, \lambda) = \begin{pmatrix} 2\lambda - 0.2 & 0 \\ 0 & 2\lambda + 2 \end{pmatrix}$$

$$x^* = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \lambda^* = 0.5$$

$$\nabla_x L(x^*, \lambda^*) = 0, \nabla_{xx} L(x^*, \lambda^*) = \begin{pmatrix} 0.8 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\nabla_x c(x^*) = \begin{pmatrix} -2x_1 \\ -2x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Critical cone $C(x, \lambda) = \{(0, w_2)^T | w_2 \in R\}$

$$w^T \nabla_{xx} L(x, \lambda) w =$$

$$\begin{pmatrix} 0 & w_2 \end{pmatrix} \begin{pmatrix} 0.8 & 0 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ w_2 \end{pmatrix} = 3w_2^2 > 0$$

3. Dual problem of LP

$$\min_x c^T x \text{ s.t. } Ax - b \geq 0$$

$$\begin{aligned} q(\lambda) &= \inf_x [c^T x - \lambda^T (Ax - b)] \\ &= \inf_x [(c - A^T \lambda)^T x + b^T \lambda)] \end{aligned}$$

$$\max_\lambda q(\lambda) = b^T \lambda \text{ s.t. } A^T \lambda = c, \lambda \geq 0$$

4. Dual problem of QP

$$\min_x \frac{1}{2}x^T G x + c^T x \text{ s.t. } Ax - b \geq 0$$

$$q(\lambda) = \inf_x \left[\frac{1}{2}x^T G x + c^T x - \lambda^T (Ax - b)\right]$$

$$\nabla_x q(\lambda) = \left[Gx + c - A^T \lambda\right] = 0$$

$$q(\lambda) = \frac{1}{2}(A^T\lambda - c)^T G^{-1}(A^T\lambda - c) + b^T\lambda$$

$$\max_\lambda q(\lambda) \quad \text{s.t. } Gx + c - A^T\lambda = 0, \lambda \geq 0$$