CS3331 Numerical Methods

Quiz 4, makeup exam

Name: __________________________, ID: __________________________

1. Prove a projection matrix $\mathbf{P} = \mathbf{I} - \mathbf{v}\mathbf{v}^T$ is idempotent ($\mathbf{PP} = \mathbf{P}$) (10pt).

$$\begin{aligned}
\mathbf{PP} &= (\mathbf{I} - \mathbf{v}\mathbf{v}^T)(\mathbf{I} - \mathbf{v}\mathbf{v}^T) \\
&= \mathbf{I} - \mathbf{v}\mathbf{v}^T - \mathbf{v}\mathbf{v}^T + \mathbf{v}\mathbf{v}^T\mathbf{v}\mathbf{v}^T \\
&= \mathbf{I} - 2\mathbf{v}\mathbf{v}^T + \mathbf{v}(\mathbf{v}^T\mathbf{v})\mathbf{v}^T \\
&= \mathbf{I} - \mathbf{v}\mathbf{v}^T = \mathbf{P}
\end{aligned}$$

2. Use Given's rotation to find an orthogonal matrix $\mathbf{Q}$ such that $\mathbf{Q}\begin{pmatrix} 12 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 13 \\ 0 \\ 0 \end{pmatrix}$. (20pt)

$$\mathbf{G}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4/5 & 3/5 \\ 0 & -3/5 & 4/5 \end{pmatrix}, \quad \mathbf{G}_1\begin{pmatrix} 12 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix}$$

$$\mathbf{G}_2 = \begin{pmatrix} 12/13 & 5/13 & 0 \\ -5/13 & 12/13 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{G}_2\begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 13 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{Q} = \mathbf{G}_2\mathbf{G}_1 = \begin{pmatrix} 12/13 & 5/13 & 0 \\ -5/13 & 12/13 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4/5 & 3/5 \\ 0 & -3/5 & 4/5 \end{pmatrix} = \begin{pmatrix} 12/13 & 4/13 & 3/13 \\ -5/13 & 48/65 & 36/65 \\ 0 & -3/5 & 4/5 \end{pmatrix}$$

3. Compute the QR decomposition of $\mathbf{A} = \begin{pmatrix} 1 & 5 & -5 \\ 2 & 0 & 5 \\ 2 & 10 & 10 \\ 4 & 0 & 0 \end{pmatrix}$. The diagonal part of the R-factor must be positive. (20pt)

$$\mathbf{Q} = \begin{pmatrix} 1/5 & 2/5 & -4/5 \\ 2/5 & -1/5 & 2/5 \\ 2/5 & 4/5 & 2/5 \\ 4/5 & -2/5 & -1/5 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 5 & 5 & 5 \\ & 10 & 5 \\ & & 10 \\ & & \end{pmatrix}$$