Homework 3: Fundamentals and Discrete Distributions

(1) It is believed that 20% of Americans do not have any health insurance. Let X equal the number with no health insurance in a random sample of $n = 15$ Americans.

(a) How is X distributed?

(b) Find the mean and variance of X.

(c) $\mathrm{P}(X \geq 2)$.

(2) Consider a random experiment of casting a pair of unbiased six-sided dice and let the r.v. X equal the *smaller* of the outcomes if they are different and the common value if they are equal.

(a) Find the p.d.f. of r.v. X.

(b) Draw a probability histogram.

(c) Find the expectation and variance of r.v. X.

(3) In a lottery, a 3-digit integer is selected at random from 000 to 999, inclusive. Let X be the integer selected on a particular day.

(a) Find the pmf (pdf) of the r.v. X.

(b) Find the mean of the r.v. X.

(c) Find variance of the r.v. X.

(4) Let the r.v. X have a Poisson discribution with the p.d.f. $f(x) = \lambda^x e^{-\lambda}/x!,\ x = 0, 1, 2, \ldots, \infty$, where $\lambda > 0$ is a known parameter.

(a) Find the mean, $E(X)$.

(b) Find the variance, Var(X).

(c) Find the mode of the probability density function f.

(5) Consider a *binomial distribution* $X \sim b(n, p)$, draw the bar chart for each density function described below.

(a) Plot the density function $X \sim b(10, 0.6)$.

(b) Plot the density function $X \sim b(9, 0.6)$.

(c) What are the modes of **(a)** and **(b)**, respectively?

(6) Let $Y \sim Poisson(\lambda)$ be a Poisson distribution with mean λ.

(a) Plot the density function $Y \sim Poisson(4)$.

(b) Plot the density function $Y \sim Poisson(7)$.

(c) What are the modes of **(a)** and **(b)**, respectively?

Partial Solutions for Homework 3, 2012

1(a) $X \sim b(15, 0.2)$.

1(b) $\mu = 3,\ \sigma^2 = 2.4$.

1(c) $P(X \geq 2) = 1 - (0.8)^15 - \begin{pmatrix} 15 \\ \\ 1 \end{pmatrix} (0.2)^1 (0.8)^{14}$.

2(a) $f(x) = \frac{13-2x}{36}$, for $x = 1, 2, 3, 4, 5, 6$

2(b) Easy bar graph.

2(c) $E[X]$=91/36, $\sigma^2 = 2555/1296 \approx 1.971$

3(a) $f(x) = 1/1000,\ 000 \leq x \leq 999$

3(b) $E[X] = 999/2 = 499.5$

3(c) $Var(X) = (10^6 - 1)/12 = 333333/4 = 83333.25$

4(a) $E[X] = \lambda$

4(b) $Var[X] = \lambda$

4(c) The mode is $\lfloor \lambda \rfloor$

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4(b) $Var[X] = \lambda$

4(c) The mode is $\lfloor \lambda \rfloor$

(5)(6)
```
% 1. Binomial Distributions
     subplot(2,2,1)
     X1=1:10;  Y1=binopdf(X1,10,0.6);
     bar(Y1)
     title('b(10,0.6)')
     subplot(2,2,2)
     X2=1:10;  Y2=binopdf(X2,9,0.6);
     bar(Y2)
     title('b(9,0.6)')
%
% 2. Poisson Distributions
     subplot(2,2,3)
     X1=1:12;  Y1=poisspdf(X1,4);
     bar(Y1)
     title('Poisson(\lambda=4)')
     subplot(2,2,4)
     X2=1:12;  Y2=poisspdf(X2,7);
     bar(Y2)
     title('Poisson(\lambda=7)')
```