

H3: Principal Component Projection

- (1) For the data set "8OX" introduced in class, there are $n = 45$ patterns from $k = 3$ categories, each pattern consists of $d = 8$ features. Each pattern can be denoted by $\mathbf{x}_i^{(k)}$, $1 \leq i \leq 15$, $1 \leq k \leq 3$, where $\mathbf{x}_i^{(k)} \in R^d$.
- (a) Compute the pooled $d \times d$ covariance matrix $C = \frac{1}{n} \sum_{k=1}^3 \sum_{i=1}^{n_k} (\mathbf{x}_i^{(k)} - \mathbf{u})(\mathbf{x}_i^{(k)} - \mathbf{u})^t$, where $\mathbf{u} = \frac{1}{n} \sum_{k=1}^3 \sum_{i=1}^{n_k} \mathbf{x}_i^{(k)}$ is the mean vector.
- (b) Report the eigenvalues $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_d$ of C .
- (c) Report the percentage of $\gamma_j = \frac{\sum_{i=1}^j \lambda_i}{\sum_{i=1}^d \lambda_i}$, $\forall 1 \leq j \leq d$.
- (d) Plot n patterns using the first two principal components.
- (e) Show the dendrogram (by complete linkage) of projected "8OX" data using only the first two principal components.
- (f) Show the dendrogram (by complete linkage) of the original "8OX" data using the $d = 8$ features.
- (2) For the data set "iris" introduced in class, there are $n = 150$ patterns from $k = 3$ categories, each pattern consists of $d = 4$ features. Repeat the same processes as required in problem (1).
- (3) For the data set "imox" introduced in class, there are $n = 192$ patterns from $k = 4$ categories, each pattern consists of $d = 8$ features. Repeat the same processes as required in problem (1).