# Mean, Variance, and Moment Function of Discrete Distributions

**Bernoulli** $f(x) = p^x(1-p)^{1-x}, \quad x = 0, 1$

$$M(t) = 1 - p + pe^t; \qquad \mu = p, \;\; \sigma^2 = p(1-p)$$

**Binomial** $f(x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$

$b(n, p) \qquad M(t) = (1 - p + pe^t)^n; \qquad \mu = np, \;\; \sigma^2 = np(1-p)$

**Geometric** $f(x) = (1-p)^{x-1}p, \quad x = 1, 2, \ldots$

$$M(t) = \frac{pe^t}{1-(1-p)e^t}, \quad t < -ln(1-p)$$

$$\mu = \frac{1}{p}, \;\; \sigma^2 = \frac{1-p}{p^2}$$

**Hypergeometric** $f(x) = \frac{C(N_1,x)C(N_2,n-x)}{C(N,n)}, \quad x \le n, x \le N_1, \, n - x \le N_2$

$$M(t) = \times$$

$$\mu = n\left(\frac{N_1}{N}\right), \;\; \sigma^2 = n\left(\frac{N_1}{N}\right)\left(\frac{N_2}{N}\right)\left(\frac{N-n}{N-1}\right)$$

**Negative Binomial** $f(x) = C(x-1, r-1)p^r(1-p)^{x-r}, \quad x = r, r+1, r+2, \ldots$

$$M(t) = \frac{(pe^t)^r}{[1-(1-p)e^t]^r} \quad t < -ln(1-p)$$

$$\mu = \frac{r}{p}, \;\; \sigma^2 = \frac{r(1-p)}{p^2}$$

**Poisson** $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$

$$M(t) = e^{\lambda(e^t-1)}; \qquad \mu = \lambda, \;\; \sigma^2 = \lambda$$

**Uniform** $f(x) = \frac{1}{m}, \quad x = 1, 2, \ldots$

$$M(t) = \frac{1}{m} \cdot \frac{e^t(1-e^{mt})}{1-e^t}; \qquad \mu = \frac{m+1}{2}, \;\; \sigma^2 = \frac{m^2-1}{12}$$

# Uniform, Exponential, Gamma, Chi-Square, Normal, Beta, Student-t, and F Distributions

**Uniform** $U(a,b)$ $f(x) = \frac{1}{b-a}$, $a \le x \le b$

$E(X) = \frac{a+b}{2}$, $Var(X) = \frac{b^2-a^2}{12}$, $M(t) = \frac{e^{tb}-e^{ta}}{t(b-a)}$, $t \ne 0$.

**Exponential** $f(x) = \frac{1}{\theta}e^{-x/\theta}$, $0 < x < \infty$

$M(t) = \frac{1}{1-\theta t}$, $t < \frac{1}{\theta}$, $E(X) = \theta$, $Var(X) = \theta^2$.

**Gamma** $f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha}x^{\alpha-1}e^{-x/\theta}$, $0 < x < \infty$, $\alpha > 0$.

$M(t) = \frac{1}{(1-\theta t)^\alpha}$, $t < \frac{1}{\theta}$, $E(X) = \alpha\theta$, $Var(X) = \alpha\theta^2$.

$\chi^2(r)$ **Chi-Square** $f(x) = \frac{1}{\Gamma(r/2)2^{r/2}}x^{(r/2)-1}e^{-x/2}$, $0 < x < \infty$

$M(t) = \frac{1}{(1-2t)^{r/2}}$, $t < \frac{1}{2}$, $E(X) = r$, $Var(X) = 2r$.

$N(\mu,\sigma^2)$ **Normal** $f(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/(2\sigma^2)}$, $-\infty < x < \infty$

$M(t) = \phi(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$, $E(X) = \mu$, $Var(X) = \sigma^2$.

**Beta Distribution** $f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$, $0 < x < 1$

$E(X) = \frac{\alpha}{\alpha+\beta}$, $Var(X) = \frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$.

Let $Z \sim N(0,1)$, $X \sim \chi^2(n)$, $Y \sim \chi^2(m)$, define $T = \frac{Z}{\sqrt{\chi^2(n)/n}}$ and $F = \frac{\chi^2(n)/n}{\chi^2(m)/m}$, then

**Student-t Distribution** $f_T(t) = \frac{\Gamma((n+1)/2)}{\sqrt{\pi n}\Gamma(n/2)[1+(t^2/n)]^{(n+1)/2}}$, $\infty < t < \infty$

**F Distribution** $f_F(w) = \frac{\Gamma((n+m)/2)(n/m)^{n/2}w^{(n/2)-1}}{\Gamma(n/2)\Gamma(m/2)[1+nw/m]^{(n+m)/2}}$, $0 < w < \infty$

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \tag{1}$$

□ $(X - \mu)/\sigma \sim N(0, 1)$

□ $Z \sim N(0, 1) \;\Rightarrow\; Y = Z^2 \sim \chi^2(1)$

When $\mu = 0$, $\sigma = 1$, $X \sim N(0, 1)$ is said to have the *standard normal distribution.* The cumulative distribution is denoted as

♢ $\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad -\infty < z < \infty$