

## Partial Solutions for Assignment 2

- (1) Let the random variable  $X$  have a p.m.f.

$$f(x) = \frac{(|x| + 1)^2}{9}, \quad x = -1, 0, 1$$

Then

- (a)  $E(X) = \sum_{x=-1}^1 xf(x) = 0$
- (b)  $E(X^2) = \sum_{x=-1}^1 x^2 f(x) = \frac{8}{9}$
- (c)  $E(3X^2 - 2X + 4) = 3E(X^2) - 2E(X) + 4 = \frac{20}{3}$

- (2) Let  $X$  have a Poisson distribution of variance 4. Then  $X \sim \text{Poisson}(\lambda = 4)$ .

- (a)  $P(X = 4) = (e^{-4}4^4)/(4!) \approx 0.1954$
- (b)  $P(2 \leq X \leq 6) = \sum_{k=2}^6 (e^{-4}4^k)/(k!) \approx 0.889 - 0.092 = 0.797$

- (3) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let  $X$  equal the number with no health insurance in a random sample of  $n = 15$  Americans.

- (a)  $X \sim b(n = 15, p = 0.2)$ .
- (b)  $E(X) = np = 3$ ,  $\text{Var}(X) = np(1 - p) = 2.4$ , and  $\text{Std.} = \sqrt{2.4} = 1.5492$ .
- (c)  $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 0.8329$ .

- (4) Let  $W$  have a geometric distribution with parameter  $p$ .

- (a)  $P(W = x) = (1 - p)^{x-1}p$ ,  $x = 1, 2, 3, \dots$ .
- (b)  $E[X] = 1/p$ ,  $\text{Var}(X) = (1 - p)/p^2$  (See lectures in class).
- (c)  $P(W > k) = \sum_{i=k+1}^{\infty} (1 - p)^{i-1}p = (1 - p)^k$ . Thus  $P(W > (k + j)|W > k) = \frac{(1-p)^{k+j}}{(1-p)^k} = P(W > j)$ . where  $k, j$  are nonnegative integers.

- (5) The probability mass functions and moment-generating functions are as follows.

- (a)  $E(X) = \frac{1}{p} = 1.25$ , then  $p = 0.8$ ,  $M(t) = \frac{0.8e^t}{1 - 0.2e^t}$ .
- (b)  $np = 30$ ,  $np(1 - p) = 12$ , then  $n = 50$ ,  $p = 0.6$ , thus  $X \sim b(50, 0.6)$ ,  $M(t) = (0.4 + 0.6e^t)^{50}$ .
- (c)  $f(x) = \frac{e^{-4}4^x}{x!}$ ,  $x = 0, 1, 2, \dots$ ,  $M(t) = e^{4(e^t - 1)}$ .

(6) Implement the following Matlab codes and print out the results.

```
%
% Script file: h2p7.m - Discrete Distributions
%
subplot(2,2,1)
X=1:10; Y=geopdf(X,0.5); bar(X,Y,0.8);
legend('Geometric Distribution: p=0.5',1)
subplot(2,2,2)
X=0:10; Y=binopdf(X,10,0.6); bar(X,Y,0.8)
legend('X \sim b(10,0.6), mode=6',1)
subplot(2,2,3)
X=0:10; Y=poisspdf(X,4); bar(X,Y,0.8)
legend('Poisson Distribution: \lambda =4',1)
subplot(2,2,4)
X=0:11; Y=binopdf(X,11,0.5); bar(X,Y,0.8)
legend('X \sim b(11,0.5), mode=5,6',2)
```

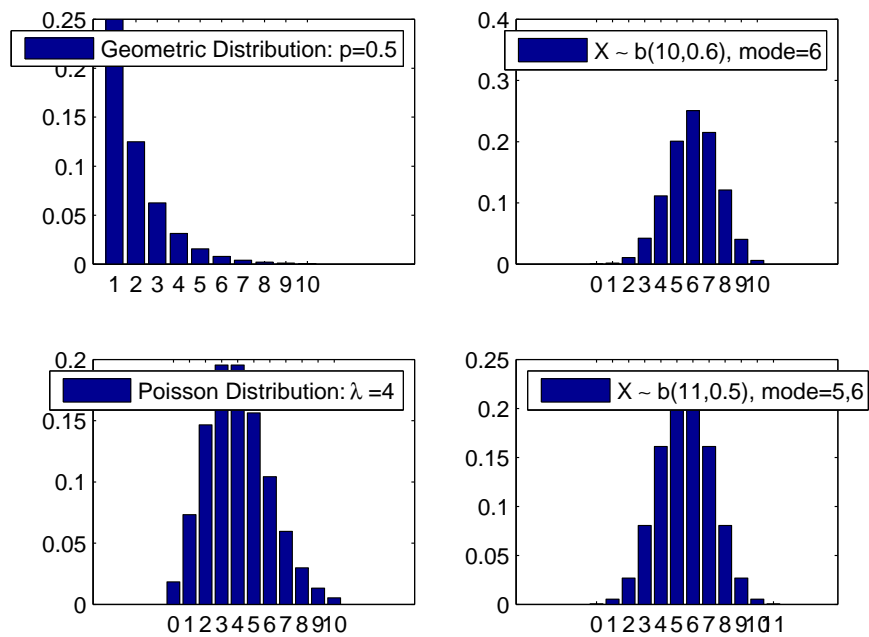


Figure 1: H2.P7 Solution.

- (7) Suppose that 2000 points are independently and randomly selected from the unit square  $S = \{(x, y) : 0 \leq x, y \leq 1\}$ . Let  $Y$  equal the number of points that fall in  $A = \{(x, y) : x^2 + y^2 \leq 1\}$ .

- (a)  $b(2000, \frac{\pi}{4})$ , i.e., binomial distribution
- (b)  $500\pi, 500\pi(1-\frac{\pi}{4})$
- (c)  $\pi$
- (d)  $\sum_{k=0}^{k=100} C(2000, k)(\pi/4)^k(1 - \pi/4)^{2000-k}$

- (8) The probability mass (density) function (p.m.f. or p.d.f.) and the space of the random variable for the problems are given as follows.

- (a)  $X \sim b(50, 0.8), f(x) = \binom{50}{x} (0.8)^x (0.2)^{50-x}, 0 \leq x \leq 50.$
- (b)  $f(x) = (0.5)^x, x = 1, 2, 3 \dots$
- (c)  $f(x) = \frac{e^{-4} 4^x}{x!}, x = 0, 1, 2, \dots$
- (d\*)  $f(x) = \frac{1}{10} e^{-x/10}, x \geq 0$
- (e\*)  $f(x) = \frac{1}{16} x^2 e^{-x/2}, x > 0.$
- (f\*)  $f(x) = \frac{1}{\sqrt{32\pi}} e^{-(x-5)^2/32}, -\infty < x < \infty$