Assignment 2

(1) Let the random variable X have a p.m.f.

$$f(x) = \frac{(|x|+1)^2}{9}, \quad x = -1,\ 0,\ 1$$

Compute (a) $E(X)$, (b) $E(X^2)$, and (c) $E(3X^2 - 2X + 4)$.

(2) Let X have a Poisson distribution of variance 4. Find (a) $\mathrm{P}(X = 4)$, (b) $\mathrm{P}(2 \leq X \leq 6)$.

(3) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let X equal the number with no health insurance in a random sample of $n = 15$ Americans.

(a) How is X distributed?

(b) Give the mean, variance, and standard deviation of X.

(c) Find $P(X \geq 2)$.

(4) Let W have a geometric distribution with parameter p.

(a) Give the probability mass function (p.m.f.) of W.

(b) Derive E(X) and Var(X).

(c) Show that $P(W > (k+j)|W > k) = P(W > j)$, where k, j are nonnegative integers.

(5) Write down the following *probability mass functions* and give their moment-generating functions.

(a) Geometric distribution with mean 1.25.

(b) Binomial distribution with mean 30 and variance 12.

(c) Poisson distribution with variance 4.

(6) Implement the following Matlab codes and print out the results.

(a) `X=1:10;  Y=geopdf(X,0.5);  bar(X,Y,0.8)`

(b) `X=0:10;  Y=binopdf(X,10,0.6);  bar(X,Y,0.8)`

(c) `X=0:10;  Y=poisspdf(X,4);  bar(X,Y,0.8)`

(7) Suppose that 2000 points are independently and randomly selected from the unit square $S = \{(x, y) : 0 \leq x, y \leq 1\}$. Let Y equal the number of points that fall in $A = \{(x, y) : x^2 + y^2 \leq 1\}$.

(a) How is Y distributed?

(b) Give the mean and variance of Y.

(c) What is the expected value of $Y/500$?

(d) What is $\mathrm{P}(Y \leq 100)$?

(8) Write down the probability mass (density) function (p.m.f. or p.d.f.) and the space of the random variable for each of the following distributions.

(a) A binomial distribution with mean 40, variance 8.

(b) A geometric distribution with mean 2.

(c) A Poisson distribution with variance 4.

(d*) An exponential distribution with mean 10.

(e*) A gamma distribution with mean 6, variance 12.

(f*) A normal distribution with mean 5 and variance 16.

(9) Let X be a discrete random variable (r.v.) having the probability mass function (p.m.f.) $f(x)$, then the mean μ, variance σ^2, and the corresponding moment-generating function $\phi(t)$ are defined as follows.

$$\mu = E[X] = \sum_x x f(x)$$

$$\sigma^2 = Var(X) = E[(X-\mu)^2] = E[X^2] - (E[X])^2$$

$$\phi(t) = E[e^{tX}] = \sum_x e^{tx} f(x)$$

Moreover, we know that

$$\mu = \phi'(0), \quad and \quad \sigma^2 = \phi''(0) - [\phi'(0)]^2$$

For a discrete type of r.v. X which has one of the following probability mass functions, **derive** the formula for the moment-generating function, and compute the mean and variance, respectively.

Binomial $f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$

Geometric $f(x) = (1-p)^{x-1} p, \quad x = 1, 2, \ldots$

Poisson $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots$

Negative Binomial (*Optional*) $f(x) = \begin{pmatrix} x-1 \\ r-1 \end{pmatrix} p^r (1-p)^{x-r}, \quad x = r, r+1, r+2, \ldots$

Hypergeometric (*Optional*) $f(x) = \dfrac{\begin{pmatrix} N \\ x \end{pmatrix}\begin{pmatrix} M \\ n-x \end{pmatrix}}{\begin{pmatrix} N+M \\ n \end{pmatrix}} \quad 0 \le x \le n, \ x \le N, \ n-x \le M$