The Weibull Distribution

The Weibull distribution is widely used for modeling the lifetimes of an electronic component (device). The nonnegative random variable $X$ with distribution function

$$F(x) = 1 - e^{-(x/\beta)^\alpha}, \ x > 0$$

$$= 0, \ x \leq 0$$

is said to have a Weibull distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$. The p.d.f. of the Weibull distribution is

$$f(x) = F'(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha}, \ x > 0$$

$$= 0, \ x \leq 0$$

Note that the exponential distribution is a special case of Weibull distribution with $\alpha = 1$.

Now the expectation and variance of a Weibull distribution could be computed by

$$E(X) = \int_0^\infty \frac{\alpha}{\beta^\alpha} t^\alpha e^{-(t/\beta)^\alpha} dt = \beta \Gamma(1 + \frac{1}{\alpha})$$

$$E(X^2) = \int_0^\infty \frac{\alpha}{\beta^\alpha} t^{\alpha+1} e^{-(t/\beta)^\alpha} dt = \beta^2 \Gamma(1 + \frac{2}{\alpha})$$

$$Var(X) = E(X^2) - (E(X))^2 = \beta^2 \left( \Gamma(1 + \frac{2}{\alpha}) - [\Gamma(1 + \frac{1}{\alpha})]^2 \right)$$

**Example:** The lifetime, measured in years, of a brand of mobile device has a Weibull distribution with parameters $\alpha = 2$ and $\beta = 13$. Compute the probability of a mobile device fails before the expiration of a two-year warranty.

**Solution:** $P(X \leq 2) = 1 - e^{[-(2/13)^2]} \approx 0.0234$. 

X=0.1:0.2:18;
a=2; b=4; Y1=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
a=2; b=7; Y2=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
a=2; b=10; Y3=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
a=2; b=13; Y4=(a/b^a)*X.^(a-1).*exp(-(X/b).^a);
plot(X,Y1,'m-',X,Y2,'g-',X,Y3,'b-',X,Y4,'r-');
legend('Weibull(2,4)','Weibull(2,7)','Weibull(2,10)','Weibull(2,13)');
title('Weibull(a,b): f(x)=(a/b^a)x^{a-1}exp[-(x/b)^a]')