(1) Let the random variable X have a p.m.f.

$$f(x) = \frac{(|x|+1)^2}{9}, \ x = -1, \ 0, \ 1$$

Compute (a) E(X), (b) $E(X^2)$, and (c) $E(3X^2 - 2X + 4)$.

- (2) Let X have a Poisson distribution of variance 4. Find (a) P(X = 4), (b) $P(2 \le X \le 6)$.
- (3) It is believed that 20% of Americans do not have any health insurance. Suppose that this is true and let X equal the number with no health insurance in a random sample of n = 15 Americans.
 - (a) How is X distributed?
 - (b) Give the mean, variance, and standard deviation of X.
 - (c) Find $P(X \ge 2)$.
- (4) Let W have a geometric distribution with parameter p.
 - (a) Give the probability mass function (p.m.f.) of W.
 - (b) Derive E(X) and Var(X).
 - (c) Show that P(W > (k + j)|W > k) = P(W > j), where k, j are nonnegative integers.
- (5) Let X be a discrete random variable (r.v.) having the probability mass function (p.m.f.) f(x), then the mean μ , variance σ^2 , and the corresponding moment-generating function $\phi(t)$ are defined as follows.

$$\mu = E[X] = \sum_{x} xf(x)$$

$$\sigma^{2} = Var(X) = E[(X - \mu)^{2}] = E[X^{2}] - (E[X])^{2}$$

$$\phi(t) = E[e^{tX}] = \sum_{x} e^{tx}f(x)$$

Moreover, we know that

$$\mu = \phi'(0), \text{ and } \sigma^2 = \phi''(0) - [\phi'(0)]^2$$

For a discrete type of r.v. X which has one of the following probability mass functions, **derive** the formula for the moment-generating function, and compute the mean and variance, respectively.

Binomial $f(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$

Geometric $f(x) = (1-p)^{x-1}p, \quad x = 1, 2, ...$

Poisson $f(x) = \frac{\lambda^{x} e^{-\lambda}}{x!}, \quad x = 0, 1, 2, ...$

Negative Binomial (Optional) $f(x) = \begin{pmatrix} x-1 \\ r-1 \end{pmatrix} p^r (1-p)^{x-r}, \quad x = r, r+1, r+2, \dots$

Hypergeometric (Optional)
$$f(x) = \frac{\binom{N}{x}\binom{M}{n-x}}{\binom{N+M}{n}} \quad 0 \le x \le n, \ x \le N, \ n-x \le M$$

- (6) Write down the following *probability mass functions* and give their moment-generating functions.
 - (a) Geometric distribution with mean 1.25.
 - (b) Binomial distribution with mean 30 and variance 12.
 - (c) Poisson distribution with variance 4.
- (7) Implement the following Matlab codes and print out the results.
 - (a) X=1:10; Y=geopdf(X,0.5); bar(X,Y,0.8)
 - (b) X=0:10; Y=binopdf(X,10,0.6); bar(X,Y,0.8)
 - (c) X=0:10; Y=poisspdf(X,4); bar(X,Y,0.8)
- (8) Suppose that 2000 points are independently and randomly selected from the unit square $S = \{(x, y) : 0 \le x, y \le 1\}$. Let Y equal the number of points that fall in $A = \{(x, y) : x^2 + y^2 \le 1\}$.
 - (a) How is Y distributed?
 - (b) Give the mean and variance of Y.

- (c) What is the expected value of Y/500?
- (d) What is $P(Y \le 100)$?
- (9) Write down the probability mass (density) function (p.m.f. or p.d.f.) and the space of the random variable for each of the following distributions.
 - (a) A binomial distribution with mean 40, variance 8.
 - (b) A geometric distribution with mean 2.
 - (c) A Poisson distribution with variance 4.
 - (d^*) An exponential distribution with mean 10.
 - (e*) A gamma distribution with mean 6, variance 12.
 - (f^*) A normal distribution with mean 5 and variance 16.