Overview

- Some related topics about regular languages
- Homework 1
Closed operations

- Union
- Concatenation
- Star
- Complement (Homework 1, Q2)

Let $A$ be a language.

$$\overline{A} = \{ x \mid x \notin A \}$$
Intersection

- Let $A$ and $B$ be languages.

Intersection:

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$
Intersection (1\textsuperscript{st} proof)

- **Theorem:** If \( A \) and \( B \) are regular languages, so is \( A \cap B \).

- **Proof**
  - By DeMorgan’s laws,
    \[
    A \cap B = \overline{A \cup B}
    \]
  - The union and complement operations are closed operations
Intersection (2\textsuperscript{nd} proof)

Example

$$A \cap B$$
Intersection (2nd proof)

- **A:** $D_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$
- **B:** $D_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$
- Construct $D = (Q, \Sigma, \delta, q, F)$
  - $Q = Q_A \times Q_B$
  - $\delta ((p,q),a) = (\delta_A(p,a), \delta_B(q,a))$
  - $q = (q_A, q_B)$
  - $F = F_A \times F_B$

$\Rightarrow$ Then we can show $L(D) = A \cap B$
We learnt that \( \{ w \mid w \text{ has equal # of } 0\text{s and } 1\text{s} \} \) is nonregular.

How about this one?

\( \{ w \mid w \text{ has equal # of } 01 \text{ and } 10 \} \)

Is it still nonregular?
Homework 1

1. DFA construction (easy)
2. Complement operation (easy)
3. NFA conversion (straightforward)
4. Pumping Lemma (easy)
5. Pumping Lemma + Closed Operation (a bit challenging)
6. A/B (hard)
5. Prove the following languages are nonregular:

(a) \( \{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\} \)

(b) \( \{wtw \mid w, t \in \{0, 1\}^+\} \)

\(^1A\) palindrome is a string such that it reads the same forward and backward. E.g., dad, level, racecar.
6. Let $A/B = \{ w \mid wx \in A \text{ for some } x \in B \}$.

(a) Suppose $A$ is recognized by
Also, suppose that $B = \{0^n1^n \mid n \geq 1\}$.

(Note: $B$ is nonregular!!)

Show $A/B$ is recognized by

![Diagram]

(start) -> 1 -> 2 -> 0

1 -> 0 -> 1

3 -> 0

3 -> 1
6.(b) In general, show that if $A$ is regular and $B$ is any language, $A/B$ is regular.
Homework 1: Further Studies

7. Reg Exp $\rightarrow$ NFA (straightforward)
8. NFA $\rightarrow$ Reg Exp (straightforward)