1. (20%) Let \( T = \{ \langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w \} \). Show that \( T \) is undecidable.

2. (15%) In the silly Post Correspondence Problem, \( SPCP \), in each pair the top string has the same length as the bottom string. Show that \( SPCP \) is decidable.

3. (20%) Show that \( A \) is Turing-recognizable if and only if \( A \leq_m A_{TM} \).

4. (20%) Show that \( A \) is decidable if and only if \( A \leq_m 0^*1^* \).

5. (25%) Let \( J = \{ w \mid \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \notin A_{TM} \} \). Show that \( A_{TM} \leq_m J \) and \( A_{TM} \leq_m \bar{J} \). Conclude that \( J \) and \( \bar{J} \) are non-Turing-recognizable.

6. (Further studies: No marks) Let \( K = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \{ \langle M \rangle \} \} \). Show that neither \( K \) nor the complement of \( K \) is Turing-recognizable.