Theory of Computation
Tutorial II

Speaker: Yu-Han Lyu
October 20, 2006
Closed operations for CFL

- Union
- Concatenation
- Star
- Reverse
- Complement is not closed
- Intersection with regular language
- Difference with regular language
- Quotient with regular language
Right Linear Grammar

• \( A \rightarrow wB \)
• \( A \rightarrow w \)
• Right Linear Grammar = Regular Language
• Left linear ?
Deterministic PDA

- NPDA $\neq$ DPDA
- DPDA $\rightarrow$ Unambiguous
- LR(k)
- How about LL, LALR, SLR??
Assignment 1

• Problem 1
  – Easiest

• Problem 2
  – Special cases: 0, 1, $\varepsilon$

• Problem 7
  – Answer is in textbook
Problem 3

- Assume there exists a pumping length $p$
- We find a string $s = w^n$, $n > p$, $n$ is prime
- No matter how we divide $s = xyz$
  - $xy|^{n+1}|z = n + n|y| = n(1 + |y|)$, which is not prime
Pumping lemma

• If L is regular language, then there exists a pumping length p, such that for any string s, \(|s|\geq p\) and \(s \in L\), we can divide s into three parts..........

• L is regular \(\rightarrow\)

\[
\{ \exists p \ : \ \forall s \ (|s|\geq p \text{ and } s \in L) \rightarrow \\
[ \exists s=xyz \ |y|>0 \text{ and } |xy|\leq p \text{ and } \forall i\geq0 \ xy^iz \in L] \}
\]
To prove L is non-regular

• L is regular →
  \{ \exists p \ \forall s \ (|s| \geq p \text{ and } s \in L) \rightarrow \\
  [ \exists s = xyz \ |y| > 0 \text{ and } |xy| \leq p \text{ and } \forall i \geq 0 \ xy^i z \in L] \}

• To prove L is non-regular, we show that: No matter what p is, we can find a string s, |s| \geq p, s \in L. But then, no matter how we divide s into xyz, at least one condition don’t hold.
To prove $L$ is consistent with pumping lemma

- $L$ is regular $\rightarrow$
  \[
  \{ \exists p \:\forall s \left( |s| \geq p \text{ and } s \in L \right) \rightarrow \]
  \[
  \left[ \exists s=xyz \mid |y| > 0 \text{ and } |xy| \leq p \text{ and } \forall _{i \geq 0} xy^i z \in L \right] \}
  \]

- We can find a $p$, such that for any string $s$, $|s| \geq p$ and $s \in L$, we can divide it into three parts...
Problem 4

• \( F=\{a^ib^jc^k \mid i, j, k \geq 0, \text{ if } i=1, \text{ then } j=k\} \)

• For any string \( s, s \in F \)
  – if \( s \) is the form \( a^ib^jc^k (i \neq 2) \)
    • \( x=\varepsilon, \ y=\text{first character, } z=\text{remainder ...} \)
  – If \( s \) is the form \( aab^jc^k (i=2) \)
    • \( x=\varepsilon, \ y=aa, z=\text{remainder ...} \)

• What is the pumping length?
Problem 5

• Alternately run in A and B
• Keep the information
  – A and B states $\rightarrow$ A x B
  – The next input will run in A or B $\rightarrow$ {Odd, Even}
Problem 6

- The same as problem 5
- In state \((a, b)\) we can
  - Run on A
  - Run on B
Problem 8

- Non-deterministically guess a final state
- Bidirectional process
  - Forward: Run $A$
  - Backward: Run $A^R$
- If end in the same state then accept
Assignment 2

• Due: 2:10 pm, October 31, 2006 (before class)
  – Late submission will not be marked
Problem 1

• Proof of pumping lemma
Problem 2

• $A = \{w \mid 2\#a(w) \neq 3\#b(w), \ w \in \{a,b\}^*\}$

• When read $a$
  – Push $a$
  – Eliminate $b$

• When read $b$
  – Push $b$
  – Eliminate $a$
Problem 3

• Let $C = \{xy \mid x, y \in \{0,1\}^* \text{ and } |x| = |y|, x \neq y\}$. Show that $C$ is a context-free language.

• At least one position is not equal

• The $i^{\text{th}}$ position of the first half is not equal to $i^{\text{th}}$ position of the second half.

• 4 variables, don’t think too difficult
Problem 4

- Let $A = \{w t w^R \mid w, t \in \{0,1\}^* \text{ and } |w| = |t|\}$. Prove that $A$ is not a context-free language.
- Pumping lemma
Problem 5

• Let $L$ be a context-free language. Then there is a constant $p$ such that for any string $z$ in $L$ with at least $p$ characters, we can mark any $p$ or more positions in $z$ to be distinguished, and then $z$ can be written as $z = uvwxy$, satisfying the following conditions:
  – (i) $vwx$ has at most $p$ distinguished positions.
  – (ii) $vx$ has at least one distinguished position.
  – (iii) For all $i \geq 0$, $u^iwx^i y$ is in $L$.

• Formal proof for all conditions
Problem 6

• Apply Ogden’s lemma and show that the language $L = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$ is inherently ambiguous.