Theory of Computation
Tutorial I

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Closed operations

- Union
- Concatenation
- Star
- Complement: \( L' = \Sigma^* - L \)
  - Final state \( \rightarrow \) non-final state
  - Non-final state \( \rightarrow \) final state
- Difference
  - \( L - M = (L' \cup M)' \)
Intersection

• If A and B are regular languages, then so is $A \cap B$

• Proof
  – Regular language is closed under complement and union operations.
  – By DeMorgan’s laws, we can use complement and union to construct intersection.
Another Proof

• Let two DFAs $D_A=(Q_A, \Sigma, \delta_A, q_A, F_A)$ and $D_B=(Q_B, \Sigma, \delta_B, q_B, F_B)$, $L(D_A)=A$, $L(D_B)=B$

• Parallel run two machines, if both accept then accept, otherwise reject.

• Formally, we construct DFA $D=(Q, \Sigma, \delta, q, F)$
  - $Q=Q_A \times Q_B$ (two tuple)
  - $F=F_A \times F_B$
  - Start state=$(q_A, q_B)$
  - $\delta((p, q), a)=(\delta_A(p, a), \delta_B(q, a))$

• Finally, we should prove $L(D) = L(A \cap B)$
Example
Reverse

- \( A^R = \{w^R \mid w \in A\} \)
- Reverse all the transitions
- Start state \( \rightarrow \) final state
- Final state(s) \( \rightarrow \) start state
- Closed
Quotient

- $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$
- Run A’s DFA
- Non-deterministically choose one state in A and guess x
Assignment 1

• Due: 3:20 pm, October 13, 2006 (before class)
  – Late submission will not be marked

• No cheating
  – Can exchange high-level idea

• Problems 1 ~ 3 are easy

• Problem 4
  – Use closed operation property to prove this language is not regular.
Problem 5

- Perfect shuffle of A and B language
  - \{w \mid w=a_1b_1\ldots a_kb_k, \text{ where } a_1\ldots a_k \in A \text{ and } b_1\ldots b_k \in B, \text{ each } a_i, b_i \in \Sigma \}

- Example
  - A={“abc”}
  - B={“def”}
  - “adbecf” \in \text{Perfect-shuffle}(A,B)
Idea

• When reading a character $a$, we should know
  – This character is in odd or even position
  – The current state in $A$ and $B$

• Problem 6 is similar
Problem 7

• Answer is in the textbook
  – After understanding, write it down in your words, otherwise..

• x and y are distinguishable by L
  – Some string z exists whereby exactly one of the strings xz and yz is a member of L

• We say that X is pairwise distinguishable by L
  – Every two distinct strings in X are distinguishable by L.

• index of L
  – Maximum number of elements in any set of strings that is pairwise distinguishable by L
Example

- This language contains at least one zero
- “01” and “00” is indistinguishable
- “11” and “01” is distinguishable by L, because “111” is not in L but “011” is in L (z=“1”)
- Index = 2
Myhill-Nerode Theorem

- L is regular if and only if it has finite index
- Application
  - Minimization DFA’s state in unique (NFA??)
  - Proof for non-regular
- Example: L={x | x is palindrome, x=x\textsuperscript{R}}
  - X={a^i | i \geq 0}
  - a and aa are distinguishable, by choosing z= ba
  - aa and aaa are distinguishable, by choosing z= baa
  - index is infinite, so this language is not regular
Problem 8

• $A_{1/2} = \{ x \mid \text{for some } y, \ |x| = |y| \text{ and } xy \in A \}$
• No hint
• Harder problem: $A_{3/3} = \{ z \mid \text{for some } x, y, \ |x| = |y| = |z| \text{ and } xyz \in A \}$
  – Can your method extend to this problem?
Reference

• Introduction to Automata Theory, Languages, and Computation (3rd Edition), by John E. Hopcroft, Rajeev Motwani, Jeffrey D. Ullman