CS2351
Data Structures

Classwork for Lecture 2
Exercise on $\Theta$ notation

Show the following bound:

1. $\sum_{k=1}^{n} k = 1+2+3+...+n = \Theta(n^2)$
Exercise on \( \Theta \) notation

Show the following bound:

\[
\sum_{k=1}^{n} k^2 = 1+4+9+\ldots+n^2 = \Theta(n^3)
\]
Exercise on $\Theta$ notation

Show the following bound:

3. $\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \ldots + \frac{1}{n} = \Theta(\log n)$
More Exercises

4. For $|c| < 1$, can you simplify

$$\sum_{k=0}^{\infty} c^k = 1 + c + c^2 + \ldots$$
More Exercises

5. For $|c| < 1$, can you show that

$$\sum_{k=1}^{\infty} kc^k = c + 2c^2 + 3c^3 + \ldots = \frac{c}{(1-c)^2}?$$
6. Can you simplify this summation?

\[ \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)} \]
Challenge

What is the name of this sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, ... ?

(a) Can you write a recursive program finding the \( n^{th} \) term? What will be the running time?

(b) Can you write a better program to improve the running time to \( O(n) \)?

(c) Can you think of an even faster way?