1. Let $S$ be a set and let $C$ be a collection of subsets of $S$. A set $S'$, with $S' \subseteq S$, is called a hitting set for $C$ if every subset in $C$ contains at least an element in $S'$. Let $\text{HITSET}$ be the language 
\[ \{ \langle C, k \rangle \mid C \text{ has a hitting set of size } k \} \]
Prove that $\text{HITSET}$ is NP-complete.

2. Let $U$ be the language 
\[ \{ \langle M, x, \#^t \rangle \mid \text{TM } M \text{ accepts input } x \text{ within } t \text{ steps on at least one branch} \} \]
Show that $U$ is NP-complete. (For this problem, you are required to prove it without using reduction from any known NP-complete problems.)

3. We say a language $A$ is in coNP if its complement, $\overline{A}$, is in NP. We call a regular expression star-free if it does not contain any star operations. Let $\text{EQ}_{\text{SF-RFX}}$ be the language 
\[ \{ \langle R, S \rangle \mid R, S \text{ are equivalent star-free regular expressions} \} \]
Show that $\text{EQ}_{\text{SF-RFX}}$ is in coNP. Why does your argument fail for general regular expressions?

4. (Choose either Q4 or Q5.) Show that the following problem is NP-complete. You are given a set of states $Q = \{ q_0, q_1, \ldots, q_\ell \}$ and a collection of pairs $\{(s_1, r_1), \ldots, (s_k, r_k)\}$ where the $s_i$ are distinct strings over $\Sigma = \{0, 1\}$, and the $r_i$ are (not necessarily distinct) members of $Q$. Determine whether a DFA $M = (Q, \Sigma, \delta, q_0, F)$ exists where $\delta(q_0, s_i) = r_i$ for each $i$. Here, the notation $\delta(q, s)$ stands for the state that $M$ enters after reading $s$, starting at state $q$. (Note that $F$ is irrelevant here).

5. (Choose either Q4 or Q5.) Consider the algorithm $\text{MINIMIZE}$, which takes a DFA $M$ as input and outputs DFA $M'$.

$\text{MINIMIZE}$ = “On input $\langle M \rangle$, where $M = (Q, \Sigma, \delta, q_0, A)$ is a DFA:
1. Remove all states of $M$ that are unreachable from the start state.
2. Construct the following undirected graph $G$ whose nodes are the states of $M$.
3. Place an edge in $G$ connecting every accept state with every nonaccept state. Add additional edges as follows.
4. Repeat until no new edges are added to $G$:
5. For every pair of distinct states $q$ and $r$ of $M$ and every $a \in \Sigma$:
6. Add the edge $(q, r)$ to $G$ if $(\delta(q, a), \delta(r, a))$ is an edge of $G$.
7. For each state $q$, let $[q]$ be the collection of states 
\[ [q] = \{ r \in Q \mid \text{no edge joins } q \text{ and } r \text{ in } G \} \].
8. Form a new DFA $M' = (Q', \Sigma, \delta', q'_0, A')$ where
   - $Q' = \{[q] | q \in Q\}$, (if $[q] = [r]$, only one of them is in $Q'$),
   - $\delta'([q], a) = [\delta(q, a)]$, for every $q \in Q$ and $a \in \Sigma$,
   - $q'_0 = [q_0]$, and
   - $A' = \{[q] | q \in A\}$.

9. Output $\langle M' \rangle$.

A. Show that $M$ and $M'$ are equivalent.

B. Show that $M'$ is minimal—that is, no DFA with fewer states recognizes the same language. You may use the Myhill-Nerode Theorem.

C. Show that $MINIMIZE$ operates in polynomial time.