1. Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

2. Let $A$ be a Turing-recognizable language consisting of descriptions of Turing machines, \{⟨$M_1$⟩, ⟨$M_2$⟩, . . .}, where every $M_i$ is a decider. Prove that some decidable language $D$ is not decided by any decider $M_i$ whose description appears in $A$.† (Hint: You may find it helpful to consider an enumerator for $A$, and re-visit the diagonalization technique.)

3. Let $E = \{⟨M⟩ \mid M$ is a DFA that accepts some string with more 1s than 0s\}. Show that $E$ is decidable. (Hint: Theorems about CFLs are helpful here.)

4. Let $C$ be a language. Prove that $C$ is Turing-recognizable if and only if a decidable language $D$ exists such that $C = \{x \mid \exists y (⟨x, y⟩ \in D)\}$.

5. (Bonus Question) Show that the problem of determining whether a CFG generates all string in $1^*$ is decidable. In other words, show that \{⟨$G$⟩ \mid $G$ is a CFG over \{0, 1\} and $1^* \subseteq L(G)$\} is a decidable language.

†The question seems strange at the first glance. In fact, it is asking you to prove that the language consisting of all descriptions of Turing deciders is not Turing-recognizable.