CS4311
Design and Analysis of Algorithms

Classwork for Lecture 2
Exercise on $\Theta$ notation

Show the following bound:

1. $\sum_{k=1 \text{ to } n} k = 1+2+3+\ldots+n = \Theta(n^2)$
Exercise on $\Theta$ notation

Show the following bound:

2. $\sum_{k=1}^{n} k^2 = 1+4+9+\ldots+n^2 = \Theta(n^3)$
Exercise on $\Theta$ notation

Show the following bound:

3. $\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \ldots + \frac{1}{n} = \Theta(\log n)$
More Exercises

4. For $|c| < 1$, can you simplify

$$\sum_{k=0}^{\infty} c^k = 1 + c + c^2 + \ldots \ ?$$
More Exercises

5. For $|c| < 1$, can you show that

$$\sum_{k=1}^{\infty} kc^k = c + 2c^2 + 3c^3 + \ldots$$

$$= \frac{c}{(1-c)^2} ?$$
More Exercises

6. Can you simplify this summation?

\[ \sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \ldots + \frac{1}{n(n+1)} \]
Challenge

What is the name of this sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, ... ?

(a) Can you write a recursive program finding the $n^{th}$ term? What will be the running time?

(b) Can you write a better program to improve the running time to $O(n)$?

(c) Can you think of an even faster way?