1. (10%) Draw the structure of the Binomial Heaps with 21 nodes.

2. (50%) Peter has maintained a set of integers using Fibonacci Heaps, and the following is its current status. The marked nodes are colored, and the minimum pointer (not shown) is pointing at 10.

Peter is going to perform the following sequence of operations. Describe clearly how the Fibonacci Heaps will change after each operation.

(i) Decrease-Key 55 to 3
(ii) Extract-Min
(iii) Insert 18
(iv) Decrease-Key 36 to 12
(v) Extract-Min

3. (40%) John has maintained a union-find data structure for a set using trees. The following is the current status of the trees, so that the set is currently partitioned into two subsets:

(a) Suppose John is using Union-By-Size strategy to perform union, and the Path-Compression heuristic to perform find. Describe what will happen after (i) union(a, x), and (ii) then find(y).

(b) Suppose John is using Union-By-Rank strategy to perform union, and the Path-Compression heuristic to perform find. Suppose the rank of x is 2 and the rank of a is 1. Describe what will happen after (i) union(a, x), and (ii) then find(y).