An Efficient Hash-Based Method for Discovering the Maximal Frequent Set

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Abstract

The association rule mining can be divided into two steps. The first step is to find out all frequent itemsets, whose occurrences are greater than or equal to the user-specified threshold. The second step is to generate reliable association rules based on all frequent itemsets found in the first step. Identifying all frequent itemsets in a large database dominates the overall performance in efficient hash-based method, HMFS, for discovering the The HMFS method maximal frequent itemsets. combines the advantages of both the DHP (Direct Hashing and Pruning) and the Pincer-Search algorithms. The combination leads to two advantages. First, the HMFS method, in general, can reduce the number of database scans. Second, the HMFS can filter the infrequent candidate itemsets and can use the filtered itemsets to find the maximal frequent itemsets. These two advantages can reduce the overall computing time of finding the maximal frequent itemsets. In addition, the HMFS method also provides an efficient mechanism to construct the maximal frequent candidate itemsets to reduce the search space. We have implemented the HMFS method along with the DHP and the Pincer-Search algorithms on a Pentium III 800 MHz PC. experimental results show that the HMFS method has better performance than the DHP and the Pincer-Search algorithms for most of test cases. In particular, our method has significant improvement over the DHP and the Pincer-Search algorithms when the size of a database is large and the length of the longest itemset is relatively long.

1. Introduction

The data mining refers to extract knowledge from a large database [10]. In recent years, data mining has attracted a growing amount of attention in the database community. One of the most important reasons is the fast growing huge amount of data that is far from the ability of human to analyze. Discovering association

rules from a database is an important technique in the data mining area. For example, for the retail business, a database may contain sales transactions. We can analyze customers buying habits through discovering the associations among products, that is, a customer who buys some products will also buy other products in the same transaction. These mined rules are very useful for the retailer to improve the marketing strategies. Other popular applications include mining web contents, mining web path traversal patterns [11], etc.

The process of mining association rules can be decomposed into two steps [13]. The first step is to find out all frequent itemsets, whose occurrences are greater than or equal to the user-specified threshold. The second step is to generate reliable association rules based on all frequent itemsets found in the first step. The cost of the first step is much more expensive than the second step. Therefore, much research focused on developing efficient algorithms for finding frequent itemsets. A well-known Apriori algorithm proposed by R. Agrawal and R. Sriank [13] was the first efficient method to find the frequent itemsets. The main contribution of the Apriori algorithm is that it utilizes the downward closure property, i.e., any superset of an infrequent itemset must be an infrequent itemset, to efficiently generate candidate itemsets for the next database scan. By scanning a database k times, the Apriori algorithm can find all frequent itemsets of a database, where k is the length of the longest frequent itemset in the database.

Many methods based on the *Apriori* algorithm have been proposed in the literature. In general, they can be classified into three categories, reduce the number of candidate itemsets, reduce the number of database scans, and the combination of bottom-up and top-down search.

• Reduce the number of candidate itemsets: Methods in this category try to generate a small number of candidate itemsets efficiently in order to reduce the computational cost. The hash-based algorithm *DHP* (Direct Hashing and Pruning) proposed by Park et al. [6] is an example. The main contribution of the DHP algorithm is that it uses a hash table to filter the huge infrequent candidate itemsets before the next database scan. However, the

DHP algorithm needs to perform database scans as many times as the length of the longest frequent itemset in a database.

- Reduce the number of database scans: Scanning a database iteratively is time consuming. Thus, methods in this category try to reduce database scans aiming at reducing disk I/O costs. The Partition algorithm proposed by Savasere et al. [1] generates all frequent itemsets with two database scans. The Partition algorithm divides the database into several blocks such that each block in the database can be fitted into the main memory and can be processed by the Apriori algorithm. However, the Partition algorithm examines much more candidate itemsets than the Apriori algorithm. Brin et al. [17] proposed the DIC algorithm that also divides the database into several blocks like the *Partition* algorithm. Unlike the Apriori algorithm, once some frequent itemsets are obtained, the DIC algorithm can generate the candidate itemsets in different blocks and then add them to count for the rest blocks. However, the DIC algorithm is very sensitive to the data distribution of a database.
- The combination of bottom-up and top-down search: Methods in this category are also based on the downward They obtain the frequent itemsets in a closure. bottom-up fashion like the Apriori algorithm. In the mean time, they use the infrequent itemsets found in the bottom-up direction to split the maximal frequent candidate itemsets in the top-down direction in each round. The advantage is that once the maximal frequent itemsets are obtained, all subsets of the maximal frequent itemsets are also identified. Therefore, all subsets of the maximal frequent itemsets do not need to be examined from the bottom-up direction. Without the top-down pruning, they need to scan database as many times as the length of the longest frequent itemset. However, the improvement is not clear when the length of the longest frequent itemset is relatively short. The Pincer-Search algorithm proposed by D. Lin et al. [2] and the MaxMiner algorithm proposed by R.J. Bayardo [6] are two examples. In these two methods, the generation of the maximal frequent candidate itemsets is not efficient. They may spend a lot of time on finding the maximal frequent itemsets.

In this paper, we propose an efficient hash-based method to generate the maximal frequent itemsets (HMFS) in the category of the combination of bottom-up and top-down search. The proposed method combines the advantages of both the DHP and the Pincer-Search algorithms. Unlike the DHP algorithm, the HMFS method is very efficient in reducing the number of database scans when the length of the longest frequent itemset is relatively long. Unlike the Pincer-Search algorithm, the HMFS method can filter the infrequent itemsets with the hash technique from the bottom-up direction and then can use the filtered itemsets to find the

maximal frequent itemsets in the top-down direction. In addition, the *HMFS* method also provides an efficient mechanism to construct the maximal frequent candidate itemsets. To evaluate the performance of the *HMFS* method, we have implemented this method along with the *DHP* and the *Pincer-Search* algorithms on a Pentium III 800 MHz PC. The experimental results show that our method has better performance than the *DHP* and the *Pincer-Search* algorithms for most of test cases. In particular, our method has significant improvement over the *DHP* and the *Pincer-Search* algorithms when the size of a database is large and the length of the longest itemset is relatively long.

The rest of the paper is organized as follows. Section 2 introduces the terminology used in this paper. The detail of the proposed *HMFS* method is presented in section 3. The experimental results of the proposed method are presented in section 4.

2. Preliminaries

In this section, we introduce some basic definitions of the association rule mining. Let $I = \{i_1, i_2, ..., i_n\}$ be a set of distinct items.

<u>Definition 1</u>: A database D is a set of transactions, where each transaction T contains a set of items in I.

<u>Definition 2</u>: A subset of I is called an *itemset*. An itemset is called a k-itemset if it contains k items.

<u>Definition 3</u>: The support of an itemset $X \subseteq I$ for a database, denoted as support(X), is the number of transactions in the database that contain all items in X.

<u>Definition 4</u>: An itemset is called a frequent itemset if its support is greater than or equal to some user-specified minimum support. Otherwise, it is an infrequent itemset. The set of all frequent k-itemsets is denoted as L_k .

<u>Definition 5</u>: Given L_{k-1} , the set of all candidate k-itemsets, C_k , is defined as $L_{k-1} \times L_{k-1} = \{X \cup Y \mid X, Y \in L_{k-1}, | X \cap Y | = k-2\}$, where k > 1.

<u>Definition 6</u>: A frequent itemset is called a maximal frequent itemset if it is not a subset of any other frequent itemsets.

<u>Definition 7</u>: An association rule is defined as $X \Rightarrow Y$, where $X, Y \subseteq I, X, Y \neq \emptyset$ and $X \cap Y = \emptyset$.

The task of the association rule mining is to discover all association rules that satisfy the minimum support and the minimum confidence. We now give an example to explain the terms described above. Consider the database shown in Table 1. Assume that $I = \{A, B, C, D, E, F\}$ and there are five transactions in the database D. Let the minimum support and the minimum confidence be 40% and 100%, respectively. All frequent itemsets are shown in Table 2. The itemsets whose support smaller than $5 \times 40\% = 2$ are infrequent itemsets. In this example, itemsets D, AB, and AE are infrequent itemsets. Note

that D, AB, and AE are the short-cut of $\{D\}$, $\{A, B\}$, and $\{A, E\}$, respectively. In this paper, all itemsets are represented by the short-cut notation. We have $L_1 = \{A, A\}$ B, C, E, F, $L_2 = \{AF, BC, BF, CE, EF, AC, BE, CF\}, L_3$ = $\{ACF, BCE, BCF, BEF, CEF\}$ and $L_4 = \{BCEF\}$. The maximal frequent itemsets are ACF and BCEF. BC \Rightarrow EF is one of the association rules that can be derived from the database shown in Table 1. Its confidence is $support(BC \cup EF)/support(BC) = 2/2 \times 100\% = 100\%.$ $BC \Rightarrow EF$ is a frequent and reliable association rule.

Table 1: Database D.

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Transaction	Items
1	A, C, D
2	B, C, E, F
3	A, B, C, E, F
4	B, E
5	A, C, F

Table 2: All frequent itemsets.

Support	Itemsets
2	AF, BC, BF, CE, EF, ACF, BCE, BCF, BEF, CEF, BCEF
3	A, B, E, F, AC, BE, CF
4	C

3. The HMFS Method

Our HMFS method combines the advantages of both the DHP and Pincer-Search algorithms. In the HMFS method, it uses the hash technique of the DHP algorithm to filter infrequent itemsets in the bottom-up direction. Then it uses a top-down technique that is similar to the Pincer-Search algorithm to find the maximal frequent The main difference of the top-down itemsets. techniques between the HMFS method and the Pincer-Search algorithm is that the HMFS method provides a more efficient mechanism to initialize the set of maximal frequent candidate itemsets than that of the Pincer-Search algorithm. By combining the advantages of the DHP and Pincer-Search algorithms, the number of database scans and the search space of items can be reduced. The algorithm of the HMFS method is given as follows.

Algorithm HMFS()

- In the first round, scan the database D to count the support of all 1-itemsets and build a hash table H_2 ;
- C_2 is constructed by $L_1 \times L_1$ and is filtered by H_2 ;
- $call\ construct_maximal_frequent_candidate_itemsets(C_2, H_2);$
- In the second round, divide D into several blocks;
- for all blocks $b \in D$ do
- Count the supports of itemsets in C_2 and MFCS;
- call $process_collision(C_2, H_2)$ to process the collisions of the hash buckets:
- Move the maximal frequent itemsets from MFCS to the hash tree;
- 9. Apply the Pincer-Search algorithm to the rest of rounds;

End_of_algorithm

```
1. C_{max} = \{x = x_1 x_2 x_3 \dots x_n \mid x_1 x_2, x_1 x_3, \dots, x_1 x_n \in C_2, \text{ where } n > 2\};
2
     m = 3; MFCS = \emptyset;
     for all x = x_1x_2x_3...x_n \in C_{max} do
       Push x into the stack initially;
        while the stack is not empty do
          Popup an element x from the stack;
6.
7.
             while m \neq n do
              k = h_2(x_i x_m), for i = 2, 3, ..., m-1; // h_2 is a hash function
             if (H_2(k) < minimum support) then
                Split x_1x_2x_3...x_n into two (n-1)-itemsets.
                      x' = x_1 x_2 x_3 ... x_i ... x_{m-1} x_{m+1} ... x_n and
                      x'' = x_1 x_2 x_3 \dots x_{i-1} x_{i+1} \dots x_m x_{m+1} \dots x_n;
11.
             if is_maximal_candidate_itemset(x") = true then
                 push x"into the stack;
12.
              else discard x",
13.
              if is_maximal_candidate_itemset(x^) = true
              then continue processing x';
14.
15
                   m++.
16.
              else x'is discarded;
17.
18.
            if (m = n \text{ and the length of } x > 2) then MFCS = MFCS \cup \{x\};
19. return MFCS:
End_of_construct_maximal_frequent_candidate_itemsets
Function is_maximal_candidate_itemset(itemset x)
     for all itemset s in the stack do
      if all items in x are also in s then return false;
      else return true;
End_of_is_maximal_candidate_itemset
Function process_collision(C_2, H_2)
    for all blocks b \in D do
       for all H_2(k) \ge \text{minimum support do}
3.
          for all c_i \in C_2 that hashed into H_2(k), where i = 1, 2, ..., n do
4.
             if (H_2(k) - \sum_{i=1}^{n} \text{support } (c_i) + \text{support } (c_j)
                 < minimum support, \forall j = 1, 2, ..., n)
              then use the infrequent 2-itemset c_i to split the itemsets in
             MFCS:
6. Remove the infrequent 2-itemset c_j from C_2;
End_of_ process_collision
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Function construct_maximal_frequent_candidate_itemsets(C2, H2)

In the algorithm HMFS(), lines 1-2 use the hash technique to filter the infrequent itemsets in C_2 in the bottom-up direction. Line 3 constructs the set of maximal frequent candidate itemsets MFCS. Line 6 counts the supports of itemsets in MFCS and C_2 . Line 7 splits the maximal frequent candidate itemsets if some conditions are satisfied. Line 8 moves the maximal frequent itemsets from MFCS to the hash tree. Line 9 performs the Pincer-Search algorithm to get the maximal frequent itemsets.

We first explain how the function construct_maximal_frequent_candidate_itemsets() works. Line 1 constructs C_{max} with all 2-itemsets that have the same first item in C_2 . Lines 3-19 generate the set of maximal frequent candidate itemsets, MFCS. generation process is as follows. Assume that an itemset x in C_{max} is denoted as $x_1x_2x_3...x_n$. Consider the first m

items in $x_1x_2x_3...x_n$, for m = 3, ..., n, and examine the 2-item subset x_ix_m of x, for i = 2, 3, ..., m-1. If the number of 2-itemsets in the corresponding hash bucket of x_ix_m is smaller than minimum support, i.e., x_ix_m is not in C_2 , split x into $x' = x_1x_2x_3...x_i...x_{m-1}x_{m+1}...x_n$ and $x'' = x_1x_2x_3...x_{i-1}x_{i+1}...x_mx_{m+1}...x_n$. Itemsets x' and x'' are then compared with elements in the stack. We have the following four cases.

Case 1. All items in x' and x'' are also in any element in the stack. Both x' and x'' are discarded. The next itemset is popped up from the stack and the generation process continues.

Case 2. Only items in x' are also in any element in the stack. Itemset x' is discarded. The generation process continues to examine $x_{i+1}x_m$ of x''.

Case 3. Only items in x'' are also in any element in the stack. Itemset x'' is discarded. The generation process continues to examine the $x_i x_{m+1}$ of x'.

Case 4. Otherwise, itemset x'' is pushed into the stack and the generation process continues to examine $x_i x_{m+1}$ of x'.

The generation process continues until m = n. Then we get a maximal frequent candidate itemset. Once one maximal frequent candidate itemset is generated, the next itemset in the stack is popped up and the generation process is applied until the stack is empty.

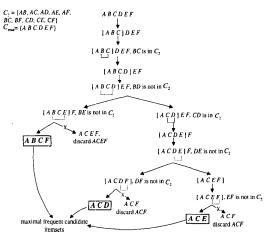


Figure 1: An example of the split process.

An example of the generation process is shown in Figure 1. Let $C_2 = \{AB, AC, AD, AE, AF, BC, BF, CD, CE, CF\}$. C_{max} is $\{ABCDEF\}$. Consider the first 3 items ABC in ABCDEF. Since BC is in C_2 , we examine ABCD in ABCDEF. Since BD is not in C_2 , ABCDEF is split into ABCEF and ACDEF. Compare ABCEF and ACDEF with elements in the stack, we have Case 4. ACDEF is pushed into the stack and the generation

process continues with ABCEF. Since BE is not in C_2 , ABCEF is split into ABCF and ACEF. Compare ABCF and ACEF with elements in the stack, we have Case 2. ACEF is discarded. A maximal frequent candidate itemset ABCF is obtained. Since the stack is not empty, itemset ACDEF is popped up from the stack and the generation process continues in a similar manner. Finally, all maximal frequent candidate itemsets, ABCF, ACD, and ACE are generated from ABCDEF.

In HMFS method, the collision of the hash buckets cannot be avoided by using the hash technique. The collision may result in an infrequent itemset be used to construct the maximal frequent candidate itemsets. For example, assume that $C_2 = \{AB, AC, AD, AE, AF, BC, BF,$ CD, CE, CF) is given. One of the maximal frequent candidate itemsets of C_2 is ABCF. Assume that AC, a frequent itemset, and AF, an infrequent itemset, are hashed into bucket 2. Since AC and AF are in the same bucket, AF cannot be filtered and will be used to construct the maximal frequent candidate itemsets. Function process_collision() provides a solution of this problem. In the following, we explain how it works. First, it divides the database into several blocks. In the second round, the supports of elements in C_2 and MFCS are counted. The number of 2-itemsets hashed into bucket kin H_2 is denoted as $H_2(k)$. Assume that there are n 2-itemsets, c_1, c_2, \ldots, c_n in C_2 , hashed into bucket k. An infrequent itemset c_i can be identified by the following

$$H_2(k) - \sum_{i=1}^{n} \operatorname{support}(c_i) + \operatorname{support}(c_j) <$$
 (1)

minimum support, $\forall j = 1, 2, ..., n$

where the supports of c_i and c_j among the k blocks are denoted as $support(c_i)$ and $support(c_j)$, respectively. In each block scanning, all infrequent itemsets in C_2 are identified and are deleted from C_2 . The identified infrequent itemsets are used to split itemsets in MFCS as well.

We now give an example to explain Equation (1). Assume that the number of transactions in a database is 10,000, the minimum support is 0.5%, and $H_2(k)$ is 100. After scanning several blocks in the database, support(AC)is 70 and support(AF) is 10. By applying Equation (1), $100-(70+10)+10 = 30 < 10,000 \times 0.5\% = 50$. Thus, we can identify AF is an infrequent itemset and AF can be discarded. The purpose of dividing a database into several blocks is that some infrequent itemsets in C_2 may be determined earlier when some blocks are scanned. The maximal frequent candidate itemsets that contain these infrequent itemsets cannot be counted further. Therefore, the division may lead us to identify those maximal frequent candidate itemsets that contain infrequent itemsets earlier and reduce the time of finding the maximal frequent itemsets.

4. Experimental results

Table 3: The meanings of all parameters

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D	Number of transactions
T	Average size of transactions
Ι	Average size of the maximal potentially large itemsets
L	Number of potentially large itemsets
N	Number of items

To evaluate the performance of the proposed method, we have implemented the HMFS method in C language along with the DHP and the Pincer-Search algorithms on a Pentium III 800 MHz PC with 512MB of main memory. The program designed by IBM Almaden Research Center is used to generate synthetic databases This program has been widely used by many researchers [1, 2, 6, 7, 8, 9, 12, 14, 17]. By setting up parameters of the program, we can generate desired databases as benchmarks to evaluate the performance of our method. Table 3 describes all the parameters used in the program. In our experiments, we set N = 1000 and L= 2000. The number of the hash buckets is 500,000. We designed two tests. In the first test, we compare the relative performance and the number of database scans for the three algorithms on four databases. The results of the first test are shown in Figure 2 and Figure 3.

Figure 2 shows the execution time of these three algorithms for test databases with various minimum supports. In Figure 2, our method is a little slower than the DHP algorithm on T10I4D100K when the minimum support is 1%. In this case, the execution time of the DHP algorithm and the HMFS method are 4 and 6 seconds, respectively. The reason is that the length of the longest itemset is two for T10I4D100K when the minimum support is 1%, i.e., only two database scans are required for T10I4D100K. The HMFS method and the DHP algorithm all require two database scans. However, the HMFS method needs to spend some time on constructing the maximal frequent candidate itemsets based on C_2 . Therefore, it takes more time than the DHPalgorithm. For other test cases, the HMFS method outperforms the DHP and the Pincer-Search algorithms. The summary reasons are given as follows.

1. In contrast with the DHP algorithm, the HMFS method finds the frequent itemsets not only in the bottom-up direction but also in the top-down direction. The execution time is improved since the number of database scans is reduced. The number of database scans is shown in Figure 3. The number of database scans required by the DHP algorithm is the length of the longest frequent itemset. In general, the number of database scans of the HMFS method is half of that of the DHP algorithm when the minimum support = 0.25% and

0.5%.

2. In contrast with the *Pincer-Search* algorithm, the *HMFS* method still has better performance than the *Pincer-Search* algorithm even though the number of database scans required by the *HMFS* method is the same as the *Pincer-Search* algorithm. There are two reasons. First, the *HMFS* method uses the hash table to filter a huge number of infrequent 2-itemsets in the C_2 instead of actually counting the supports of all 2-itemsets. Second, it constructs the maximal frequent candidate itemsets by using the hash technique instead of the combination of all distinct 1-itemsets in a database. The search space is reduced substantially.

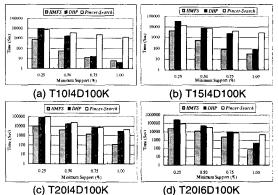


Figure 2: The execution time of the *HMFS* method, *DHP*, and *Pincer-Search* algorithms on various test databases with increasing minimum supports.

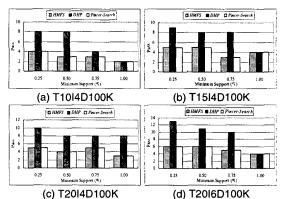


Figure 3: The number of database scans of the *HMFS* method, *DHP*, and *Pincer-Search* algorithms on various test databases with increasing minimum supports.

In the second test, we evaluate the performance of the *HMFS* method and the *DHP* algorithm on the test databases with various database sizes. The results of the second test are shown in Figure 4. The performance of the *Pincer-Search* algorithm is not included since it takes too much time to get the execution results for the test databases. In Figure 4, the number of transactions in the test databases is set from 100K to 500K and the minimum support is 0.75%. From Figure 4, we can see that both the execution time of *HMFS* and *DHP* increases when the number of transactions increases. However, the execution time of the *DHP* algorithm is near linear to the size of test databases. The *HMFS* method is not so sensitive to the size of a database compared to the *DHP* algorithm. Therefore, our *HMFS* method performs much better when the database size is larger.

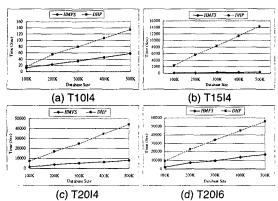


Figure 4: The execution time of the *HMFS* method and the *DHP* algorithm on the test databases with various database sizes. (Minimum Support ≈ 0.75%)

5. Conclusions

In this paper, we have proposed an efficient hash-based method, HMFS, for discovering the maximal frequent itemsets. The method combines the advantages of the DHP and the Pincer-Search algorithms. combination leads to two advantages. First, the HMFS method, in general, can reduce the number of database scans. Second, the HMFS method can filter infrequent itemsets and use the filtered itemsets to find the maximal frequent itemsets faster. In addition, an efficient mechanism to construct the maximal frequent candidate itemsets is provided. To evaluate the performance of our method, we have implemented the proposed method along with the DHP and the Pincer-Search algorithms on a Pentium III 800 MHz PC. The experiments were conducted on various benchmark databases. experimental results show that our method has better performance than the DHP and the Pincer-Search algorithms for most of test cases. In particular, our method has significant improvement over the DHP and the Pincer-Search algorithms when the size of a database is large and the length of the longest itemset is relatively long.

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