Tutorial 3
Theory of Computation

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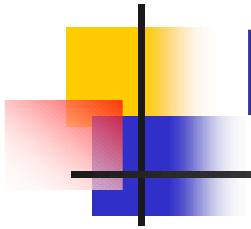
11/23



Homework 3

- We have 5 questions this time:

Q1: Very Easy

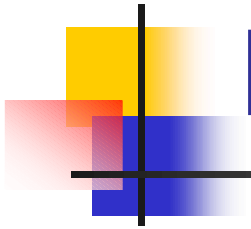
Q2: Easy

Q3: Easy

Q4: Moderate

Q5: Easy/Moderate

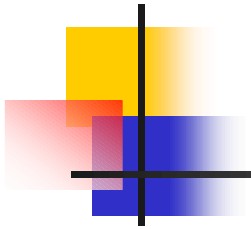
Q6: (Further studies): Hard to Think



Homework 3

1. Let *k*-PDA be a pushdown automaton that has *k* stacks
   - Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA.

- We already know that 1-PDAs are more powerful than 0-PDAs (why?)

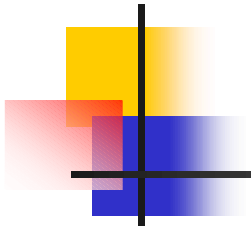


(a) Show that some language can be recognized by a 2-PDA but not a 1-PDA.

Hint:

Find a (simple) non-CFL that can be recognized by 2-PDA

- Conclude that 2-PDAs are more powerful than 1-PDAs (How?)

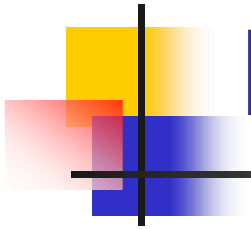


(b) (Further studies)

Show that if L can be recognized by a 3-PDA, L can be recognized by some 2-PDA

(Hint: use some kind of encoding)

➔ If the above is true, we can conclude that 2-PDAs are as powerful as 3-PDAs (why?)



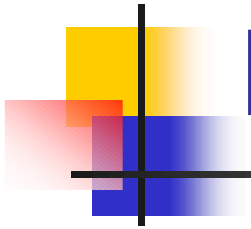
Homework 3

2. Show that:

L is decidable

if and only if

some enumerator enumerates L in
****lexicographic order****



Homework 3

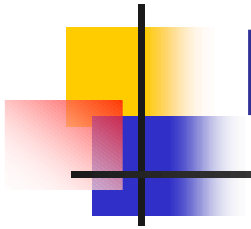
3. Let $S = \{<M> |$ M is a DFA that accepts w whenever it accepts w^R $\}$

Show that S is decidable.

Hint:

If M recognizes L, can we find an NFA N that recognizes L', where $L' = \{ w^R \mid w \text{ is in } L \}$?

If M and N are found. Can we decide if M is in S?



Homework 3

4. Let PAL_{DFA} = {<M>| M is a DFA that accepts some palindrome}

Show that PAL_{DFA} is decidable.

Hint:

(i) Fact: CFL ∩ Reg → CFL (Prob 2.18)

(ii) Prob 4.23 shows how to prove a similar language is decidable

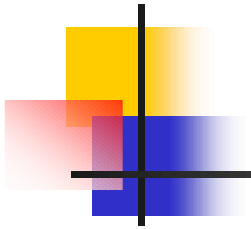
Homework 3

- 5. Suppose that we have a decider D such that D decides if the language of a CFG is infinite. That is,

  D is a decider for the language:

  $INFINITE_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) \text{ is infinite}\}$.

BTW, does D exist?



By using D or otherwise, show that

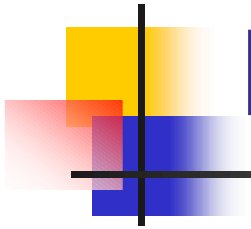
$C_{CFG} = \{<G, k> |$ G is a CFG and $L(G)$ contains exactly k strings where $k \geq 0$ or $k = \infty\}$

is decidable.

Hint :

Let p be the pumping length of G.

If $L(G)$ is finite, $L(G)$ cannot have any string longer than p (why?)



Homework 3

6. (Further studies)

Prove that:

C is Turing-recognizable

if and only if

a decidable language D exists

such that $C = \{x \mid \exists y\, (\langle x, y \rangle \in D)\}$.