Tutorial 2
Theory of Computation

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Overview

- Pumping lemma
- Homework 2

Review of Pumping Lemma

- If A is a regular language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into three pieces, s=xyz, satisfying the following conditions:
  - 1. for each $i \geqq 0$, $xy^iz \in A$
  - 2. $|y| > 0$, and
  - 3. $|xy| \leqq p$

Example (What's wrong?)

- Prove that the language $B = \{0^n 1^n | n \geqq 0\}$ is not regular
- Proof:
- Consider $s = 0^p 1^p = xyz$ ← what is p?
- So, if $y = 0$, $xyyz = 0^{p+1} 1^p$ which is not in B
- Thus, by pumping lemma, B is non-regular

Example (What's wrong?)

- Prove that the language $B = \{0^n1^n | n \geq 0\}$ is not regular

why B has pumping length??

- Proof:
- Let p be the pumping length of B
- Consider $s = 0^p1^p = xyz$
- So, if $y = 0$, $xyyz = 0^{p+1}1^p$ which is not in B
- Thus, by pumping lemma, B is non-regular

Example (What's wrong?)

- Prove that the language $B = \{0^n 1^n | n \geqq 0\}$ is not regular
- Proof:
- Assume B is regular. Let p be the pumping length of B
- Consider $s = 0^p 1^p = xyz$ ⟵ what is *xyz*?
- So, if $y = 0$, $xyyz = 0^{p+1} 1^p$ which is not in B
- Thus, by pumping lemma, B is non-regular

Example (What's wrong?)

- Prove that the language $B = \{0^n 1^n \mid n \geqq 0\}$ is not regular
- Proof:
- Assume B is regular. Let p be the pumping length of B. Consider $s = 0^p 1^p$.
- Let s be divided into 3 parts such that $s = xyz$
- So, if $y = 0$, $xyyz = 0^{p+1} 1^p$ which is not in B
- Thus, by pumping lemma, B is non-regular

y=0? How about other cases of y?

Example (What's wrong?)

- Assume B is regular. Let p be the pumping length of B. Consider $s = 0^p 1^p$.
- Let s be divided into 3 parts such that $s = xyz$
- If $y = 0^k$ for some $1 \leqq k \leqq p$, then $xyyz = 0^{p+k}1^p$ which is not in B
- Thus, by pumping lemma, B is non-regular

y=0^k? How about other cases of y?
Currently, y can be any substring of s

Example (Good enough?)

- Assume B is regular. Let p be the pumping length of B. Consider $s = 0^p1^p$.
- Let s be divided into 3 parts such that $s = xyz$ with $|y| > 0$, $|xy| \leq p$
- So, y must be 0^k for some $1 \leq k \leq p$. Then $xyyz = 0^{p+k}1^p$ which is not in B
- Thus, by pumping lemma, B is non-regular

Almost perfect… But we must show s is in B to apply pumping lemma!

Example (Perfect Proof)

- Assume B is regular. Let p be the pumping length of B. Consider $s = 0^p1^p$, which is obviously in B, and $|s|$ is at least p.
- Let s be divided into 3 parts such that $s = xyz$ with $|y| > 0$, $|xy| \leq p$
- So, y must be 0^k for some $1 \leq k \leq p$. Then $xyyz = 0^{p+k}1^p$ which is not in B
- Thus, by pumping lemma, we observe a contradiction
- Thus, we conclude that B is non-regular

Homework 2

- 1. Completing a proof (Easy)
- 2. Finding CFG (Moderate)
- 3. CFG → CNF (Straightforward)
- 4. Finding CFG or PDA (Hard)
- 5. Pumping lemma (Easy)

Question 2(b)

Find CFG for:

$\{x_1\#x_2\#\ldots\#x_k \mid k \geqq 1$, each $x_i \in \{a, b\}^*$, and for some i and j, $x_i = x_j^R\}$

Attention:

We need to allow for the case when $i=j$.

That is, some x_i is a palindrome. Also, ε is in the language since it is a palindrome.

Question 4

Let $C = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } x \neq y\}$.

Show that C is a context-free language.

Hint: We can find CFG or PDA for this.

One observation is that: if s is in C, either

Case 1. $|x| \neq |y|$ (easy to generate)

or Case 2. The i^{th} char of x is different from the i^{th} char of y (need thinking)

Homework 2: Further Studies

- 6. Properties of CFG
- 7. Application of Question 6
- 8. Proving Non-CFG (Hard)

- Thank you