Tutorial 2
Theory of Computation

呂紹甲
(Lu Shao-chia)
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Overview

- Pumping lemma
- Homework 2
Review of Pumping Lemma

- If $A$ is a regular language, then there is a number $p$ (the pumping length) where, if $s$ is any string in $A$ of length at least $p$, then $s$ may be divided into three pieces, $s=xyz$, satisfying the following conditions:
  
  1. for each $i \geq 0$, $xy^i z \in A$
  2. $|y| > 0$, and
  3. $|xy| \leq p$
Example (What’s wrong?)

- Prove that the language $B = \{0^n1^n | n \geq 0\}$ is not regular
- Proof:
  - Consider $s = 0^p1^p = xyz$  \(\text{what is } p?\)
  - So, if $y = 0$, $xyyz = 0^{p+1}1^p$ which is not in $B$
  - Thus, by pumping lemma, $B$ is non-regular
Example (What’s wrong?)

- Prove that the language $B = \{0^n1^n | n \geq 0\}$ is not regular
- Proof:
  - Let $p$ be the pumping length of $B$
  - Consider $s = 0^p1^p = xyz$
  - So, if $y = 0$, $xyyz = 0^{p+1}1^p$ which is not in $B$
  - Thus, by pumping lemma, $B$ is non-regular

why $B$ has pumping length??
Example (What’s wrong?)

- Prove that the language $B = \{0^n1^n | n \geq 0\}$ is not regular

Proof:
- Assume $B$ is regular. Let $p$ be the pumping length of $B$
- Consider $s = 0^p1^p = xyz \quad \text{what is } xyz$?
- So, if $y = 0$, $xyyz = 0^{p+1}1^p$ which is not in $B$
- Thus, by pumping lemma, $B$ is non-regular
Example (What’s wrong?)

- Prove that the language \( B = \{0^n1^n | n \geq 0 \} \) is not regular
- Proof:
  - Assume \( B \) is regular. Let \( p \) be the pumping length of \( B \). Consider \( s = 0^p1^p \).
  - Let \( s \) be divided into 3 parts such that \( s = xyz \)
  - So, if \( y = 0 \), \( xyyz = 0^{p+1}1^p \) which is not in \( B \)
  - Thus, by pumping lemma, \( B \) is non-regular

\( y = 0? \) How about other cases of \( y? \)
Example (What’s wrong?)

- Assume $B$ is regular. Let $p$ be the pumping length of $B$. Consider $s = 0^p1^p$.
- Let $s$ be divided into 3 parts such that $s = xyz$.
- If $y = 0^k$ for some $1 \leq k \leq p$, then $xyyz = 0^{p+k}1^p$ which is not in $B$.
- Thus, by pumping lemma, $B$ is non-regular.

$y=0^k$? How about other cases of $y$? Currently, $y$ can be any substring of $s$. 
Example (Good enough?)

- Assume $B$ is regular. Let $p$ be the pumping length of $B$. Consider $s = 0^p1^p$.
- Let $s$ be divided into 3 parts such that $s = xyz$ with $|y| > 0$, $|xy| \leq p$
- So, $y$ must be $0^k$ for some $1 \leq k \leq p$. Then $xxyyz = 0^{p+k}1^p$ which is not in $B$
- Thus, by pumping lemma, $B$ is non-regular

Almost perfect... But we must show $s$ is in $B$ to apply pumping lemma!
Example (Perfect Proof)

- Assume $B$ is regular. Let $p$ be the pumping length of $B$. Consider $s = 0^p1^p$, which is obviously in $B$, and $|s|$ is at least $p$.
- Let $s$ be divided into 3 parts such that $s = xyz$ with $|y| > 0$, $|xy| \leq p$.
- So, $y$ must be $0^k$ for some $1 \leq k \leq p$. Then $xyyz = 0^{p+k}1^p$ which is not in $B$.
- Thus, by pumping lemma, we observe a contradiction.
- Thus, we conclude that $B$ is non-regular.
Homework 2

- 1. Completing a proof (Easy)
- 2. Finding CFG (Moderate)
- 3. CFG → CNF (Straightforward)
- 4. Finding CFG or PDA (Hard)
- 5. Pumping lemma (Easy)
Question 2(b)

Find CFG for:

\[ \{ x_1 \# x_2 \# \ldots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R \} \]

Attention:

We need to allow for the case when \( i = j \).
That is, some \( x_i \) is a palindrome. Also, \( \varepsilon \) is in the language since it is a palindrome.
Question 4

Let $C = \{ x \# y \mid x, y \in \{0,1\}^* \text{ and } x \neq y \}$. Show that $C$ is a context-free language.

Hint: We can find CFG or PDA for this.

One observation is that: if $s$ is in $C$, either

Case 1. $|x| \neq |y|$ (easy to generate)

or Case 2. The $i$th char of $x$ is different from the $i$th char of $y$ (need thinking)
Homework 2: Further Studies

- 6. Properties of CFG
- 7. Application of Question 6
- 8. Proving Non-CFG (Hard)
Thank you