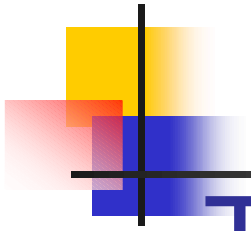

Tutorial 1
Theory of Computation

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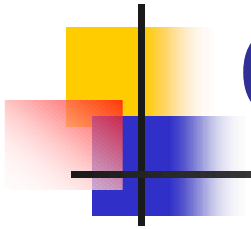
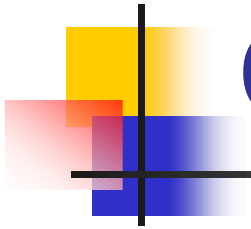


Overview

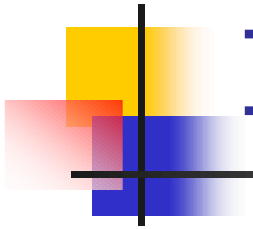
- Some related topics about regular languages
- Homework 1



Closed operations

- Union
- Concatenation
- Star
- Complement (Homework 1, Q2)
  - Let A be a language.

$$\overline{A} = \{x \mid x \notin A\}$$

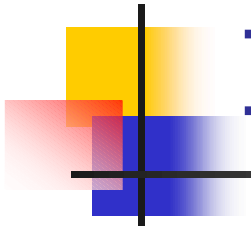


Intersection

- Let A and B be languages.

  Intersection :

  $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$



Intersection (1st proof)

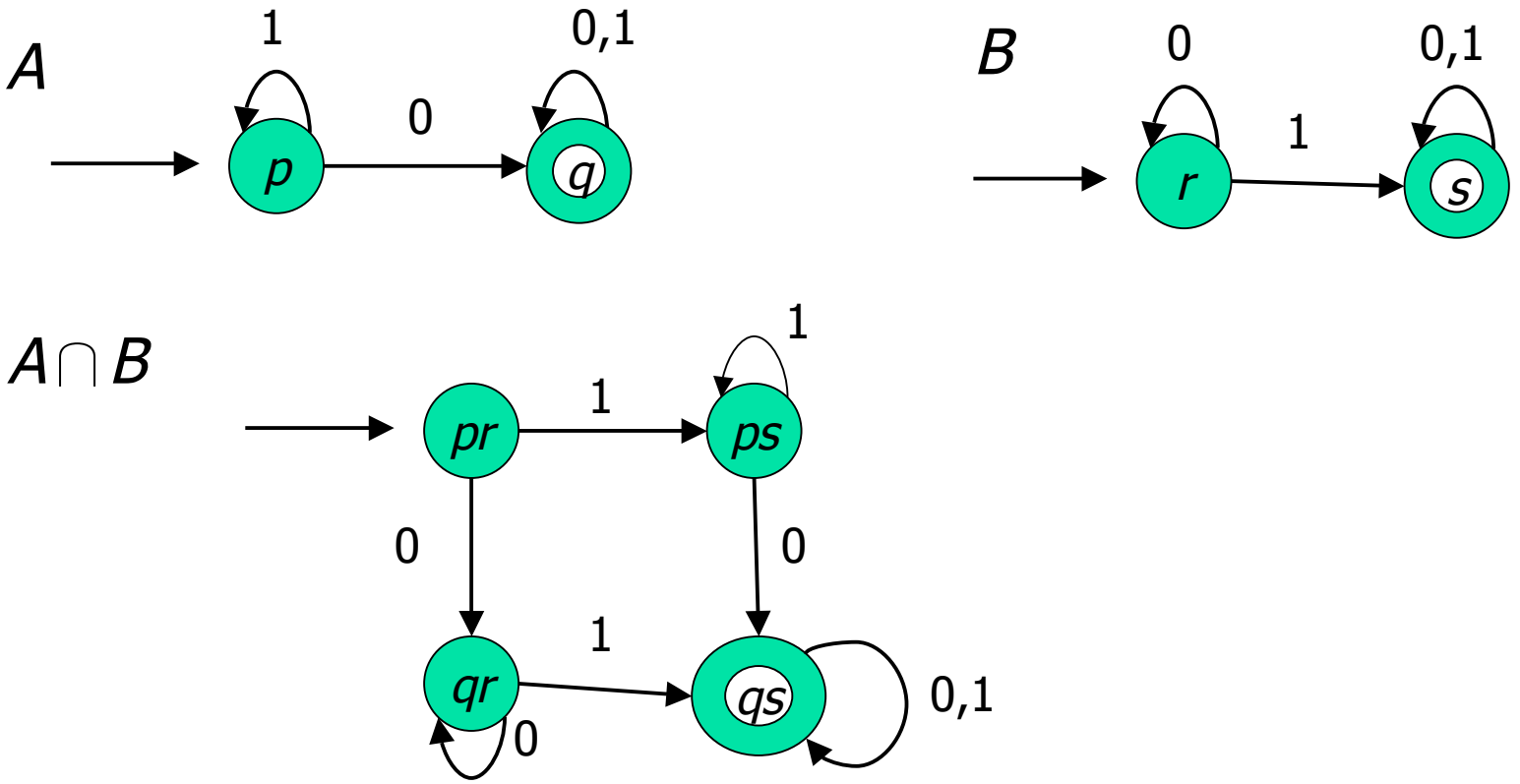
- Theorem: If A and B are regular languages, so is $A \cap B$.
- Proof
  - By DeMorgan's laws,

$$A \cap B = \overline{\overline{A} \cup \overline{B}}$$

  - The union and complement operations are closed operations

Intersection (2nd proof)

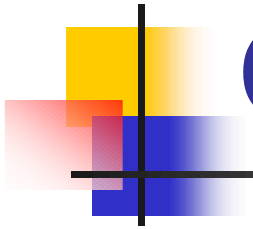
Example



Intersection (2nd proof)

- A: $D_A = (Q_A, \Sigma, \delta_A, q_A, F_A)$
- B: $D_B = (Q_B, \Sigma, \delta_B, q_B, F_B)$
- Construct $D = (Q, \Sigma, \delta, q, F)$
  - $Q = Q_A \times Q_B$
  - $\delta((p,q),a) = (\delta_A(p,a), \delta_B(q,a))$
  - $q = (q_A, q_B)$
  - F=$F_A \times F_B$

➔ Then we can show L(D)= $A \cap B$



Quiz

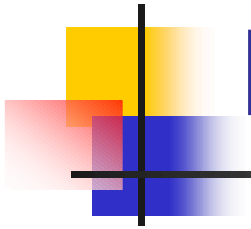
- We learnt that

  $\{w \mid w$ has equal # of 0s and 1s$\}$

  is nonregular
- How about this one?

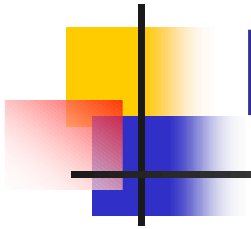
  $\{w \mid w$ has equal # of 01 and 10 $\}$

  Is it still nonregular?



Homework 1

1. DFA construction (easy)
2. Complement operation (easy)
3. NFA conversion (straightforward)
4. Pumping Lemma (easy)
5. Pumping Lemma + Closed Operation (a bit challenging)
6. A/B (hard)



Homework 1

5. Prove the following languages are nonregular:

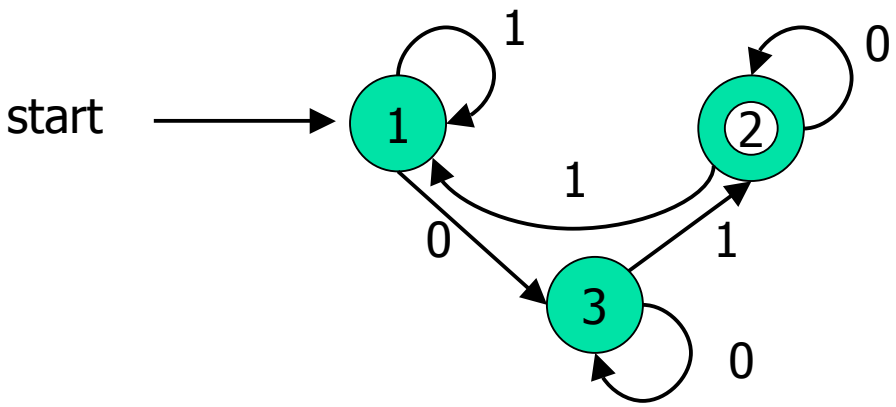
   (a) $\{w \mid w \in \{0, 1\}^*$ is not a palindrome$\}$[1]
   (b) $\{wtw \mid w, t \in \{0, 1\}^+\}$

[1]A palindrome is a string such that it reads the same forward and backward. E.g., dad, level, racecar.

Homework 1

6. Let A/B = $\{w \mid wx \in A \text{ for some } x \in B\}$.

(a) Suppose A is recognized by

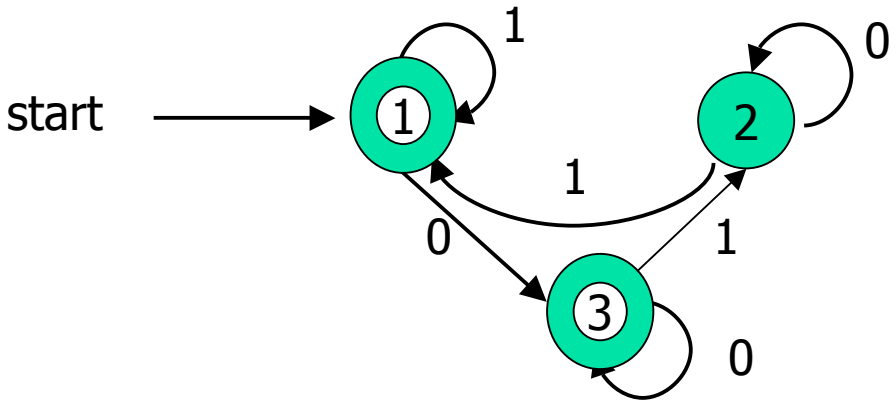


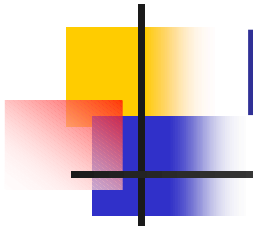
Homework 1

Also, suppose that $B = \{0^n 1^n \mid n \geq 1\}$.

(Note: B is nonregular!!)

Show A/B is recognized by

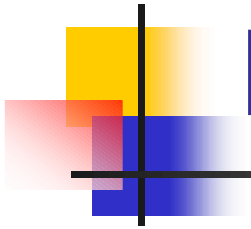




Homework 1

6.(b) In general, show that

if A is regular and B is any language,
A/B is regular.



Homework 1: Further Studies

7. Reg Exp → NFA (straightforward)
8. NFA → Reg Exp (straightforward)