CS5371 Theory of Computation Last (but not least) Lecture: The Revision

- Various mathematical models (such as DFA/NFA/PDA) that can perform string decision problems
- Various mathematical expressions (such as RE/CFG) that can generate strings
- DFA = NFA = RE (regular languages)
- PDA = CFG (context-free languages)

- How to show a language is regular
  - Give a DFA/NFA, Write a RE
  - E.g., A = { even-length string ending with 00 }
- · How to show a language not regular
  - Pumping Lemma (pumping length p, xy<sup>k</sup>z)
  - E.g., B = { palindrome }
  - E.g.,  $C = \{ O \times 1^{y} | x < y \}$

- · How to show a language context-free
  - Give a PDA, Write a CFG
  - E.g., D = { palindrome }
- How to show a language not context-free
  - Pumping Lemma (pumping length p, uv<sup>k</sup>xy<sup>k</sup>z)
  - E.g., E = { 0×1×2× }
  - E.g., F = { ww }

- Stronger mathematical models (such as DTM/NTM/Enumerator) that can solve more string decision problems
- DTM = NTM (in deciding/recognizing power)
- DTM = Enumerator (in recognizing power)
- Decidable Language
  - $A_{DFA}$ ,  $E_{DFA}$ ,  $E_{QDFA}$ ,  $A_{CFG}$ ,  $E_{CFG}$ , ...
- Recognizable Language
  - A<sub>TM</sub>, HALT<sub>TM</sub>, ...

- How to show a language recognizable
  - Give a TM recognizer
  - Finite steps to Accept
  - May Loop if Not Accept
- How to show a language decidable
  - Give a TM decider
  - Finite steps to Accept and to Reject

- How to show a language undecidable
  - Diagonalization Proof (E.g.,  $A_{TM}$ )
  - Reduction Proof
  - E.g., HALT<sub>TM</sub>,  $E_{TM}$ , Rice,  $EQ_{TM}$ , Post,  $E_{LBA}$ , ...
- Some language and its complement are both non-recognizable
  - Mapping Reduction Proof
  - E.g., EQ<sub>TM</sub> (by Reduction from  $A_{TM}$ ')

- Decidable = Deciding in Finite Steps
- Finite is TOO LARGE
  - Measuring Time Complexities
- Relationship among models
- 1-tape DTM vs k-tape DTM
- · DTM vs NTM

- P = Deciding in Polynomial Time by DTM
  E.g, PATH, RELPRIME
- NP = Verifying in Polynomial Time by DTM
   = Deciding in Polynomial Time by NTM
  - E.g., SAT, COMPOSITES, HAMPATH, ...
- Some problems in NP are the hardest (NP-complete)
  - E.g., SAT (Cook-Levin), 3SAT, CLIQUE, ...

- How to show a language is in NP
  - Give a DTM verifier, or
  - Give an NTM decider
  - Show that running time is polynomial (in terms of input length)
- How to show a language NP-complete
  - Show that it is in NP
  - Show that every NP problem can be reduced to it in polynomial time
  - Polynomial Time (Mapping) Reduction Proof

## About the Exam

- Jan 11, 2008 (next Friday) Don't Forget!!!
- Venue: This Room
- Time: 3:20pm 6:20pm
- Format:
  - Around 7 Questions
  - Easy to Moderately Difficult
  - Most from Notes/HW, Some Unseen
- Tentative marks will be sent by email within one week
- Finalized after that

# Acknowledgement

- Thanks all of you for choosing this course
- Special thanks to those who have sent me comments and suggestions, and those who have observed the bugs in the Notes/HW
- Thanks Shao-Chia (紹甲) for being a very responsible tutor
- The textbook is also wonderful
- Advertisement: Algo, Randomized Algo

GOOD LUCK in the EXAM