CS5371 Theory of Computation

Homework 5

Due: 3:20 pm, January 4, 2007 (before class)

1. (25%) Prove that if $P = NP$, then *PATH* is NP-complete.[1]

2. (25%) Let *LPATH* denote the language:

   $$LPATH = \{\langle G, s, t, k\rangle \mid G \text{ contains a simple path of length at least } k \text{ from } s \text{ to } t\}.$$

   Show that *LPATH* is NP-complete. (Hint: Reduction from *HAMPATH*.)

3. Let ϕ be a cnf-formula. An assignment to the variables of ϕ is called *not-all-equal* if in each clause, at least one literal is TRUE and at least one literal is FALSE.

   Let $\neq SAT$ be the language:

   $$\neq SAT = \{\langle \phi \rangle \mid \phi \text{ is a cnf-formula which has a satisfying not-all-equal assignment}\}.$$

   For example,

   - $\phi_1 = (u \vee v) \wedge (v \vee x)$ is in $\neq SAT$;
   - $\phi_2 = (u \vee v) \wedge (\neg u \vee v)$ is not in $\neq SAT$.

   (25%) Show that $\neq SAT$ is NP-complete.

   Hint: Reduction from *CNF-SAT* by replacing each clause C_i

   $$(x_1 \vee x_2 \vee \cdots \vee x_k)$$

   with the two clauses

   $$(x_1 \vee x_2 \vee \cdots \vee x_{k-1} \vee z_i) \quad \text{and} \quad (\neg z_i \vee x_k)$$

4. (25%) Let S be a finite set and $C = \{C_1, C_2, \ldots, C_k\}$ be a collection of subsets of S, for some $k > 0$. We say S is *two-colorable* with respect to C if we can color the elements of S in either red or blue, such that each subset C_i contains at least a red element and at least a blue element.

   Let *2COLOR* denote the language:

   $$2COLOR = \{\langle S, C\rangle \mid S \text{ is two-colorable with respect to } C\}.$$

   Show that *2COLOR* is NP-complete. (Reduction from which NP-complete problem??)

5. (Further Studies: No marks) If $P = NP$, will all languages in P become NP-complete?

6. (Further Studies: No marks) Let $CNF_k = \{\langle\phi\rangle \mid \phi$ is a satisfiable cnf-formula where each variable appears in at most k places$\}$.

   (a) Show that $CNF_2 \in P$.

   (b) Show that CNF_3 is NP-complete.

[1] An immediate corollary of this: If *PATH* is not NP-complete, then $P \neq NP$.