

CS5371 THEORY OF COMPUTATION

Homework 4

Due: 3:20 pm, December 21, 2007 (before class)

1. (20%) Let $T = \{\langle M \rangle \mid M \text{ is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable.
2. (15%) In the *silly Post Correspondence Problem*, $SPCP$, in each pair the top string has the same length as the bottom string. Show that $SPCP$ is decidable.
3. (20%) Show that A is Turing-recognizable if and only if $A \leq_m A_{TM}$.
4. (20%) Show that A is decidable if and only if $A \leq_m 0^*1^*$.
5. (25%) Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \notin A_{TM}\}$. Show that $A_{TM} \leq_m J$ and $A_{TM} \leq_m \bar{J}$. Conclude that J and \bar{J} are non-Turing-recognizable.
6. (Further studies: No marks) Let $K = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \{\langle M \rangle\}\}$. Show that neither K nor the complement of K is Turing-recognizable.