CS5371 Theory of Computation

Homework 3

Due: 2:10 pm, December 4, 2007 (before class)

1. Let k-PDA be a pushdown automaton that has k stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. We already know that 1-PDAs are more powerful than 0-PDAs (since 1-PDAs recognize a larger class of languages).

   (a) (15%) Show that some language can be recognized by a 2-PDA but not a 1-PDA. Conclude that 2-PDAs are more powerful than 1-PDAs.

   (b) (Further studies: No marks) Show that if a language L can be recognized by a 3-PDA, L can be recognized by some 2-PDA. Conclude that 2-PDAs are as powerful as 3-PDAs.

2. (20%) Show that a language is decidable if and only if some enumerator enumerates the language in a way that shorter strings are enumerated first, while for equal-length strings, they are enumerated in lexicographic order.

3. Let $S = \{\langle M \rangle \mid M$ is a DFA that accepts w whenever it accepts the reverse of $w\}$.

   (a) (5%) Give an example of a DFA that is in S.

   (b) (20%) Show that S is decidable.

4. (20%) Let $PAL_{DFA} = \{\langle M \rangle \mid M$ is a DFA that accepts some palindrome$\}$. Show that PAL_{DFA} is decidable. (Hint: Prob 2.18 and Prob 4.23 are helpful here.)

5. (20%) Suppose that we have a decider D that decides if the language of a CFG is infinite. That is, D is a decider for the language:

$$INFINITE_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) \text{ is infinite}\}.$$

   By using D or otherwise, show that the following language:

   $C_{CFG} = \{\langle G, k \rangle \mid G$ is a CFG and $L(G)$ contains exactly k strings where $k \geq 0$ or $k = \infty\}$

   is decidable.

6. (Further studies: No marks) Let C be a language. Prove that C is Turing-recognizable if and only if a decidable language D exists such that $C = \{x \mid \exists y(\langle x, y \rangle \in D)\}$.