CS5371 Theory of Computation

Homework 2
Due: 2:10 pm, October 30, 2006 (before class)

1. (10%) Complete the proof of the theorem in Lecture 7, pages 17–18.
2. (30%) Give the CFGs generating the following languages:
   (a) The complement of the language $\{\mathtt{a}^n\mathtt{b}^n \mid n \geq 0\}$
   (b) $\{x_1\#x_2\#\cdots\#x_k \mid k \geq 1,$ each $x_i \in \{\mathtt{a},\mathtt{b}\}^*,$ and for some i and $j, x_i = x_j^R\}$
3. (15%) Convert the following CFG into an equivalent CFG in Chomsky normal form: [1]

$$\begin{aligned} A &\rightarrow BAB \mid B \mid \varepsilon \\ B &\rightarrow \mathtt{00} \mid \varepsilon \end{aligned}$$

4. (15%) Let $C = \{x\#y \mid x, y \in \{\mathtt{0},\mathtt{1}\}^*$ and $x \neq y\}$. Show that C is a context-free language.
5. (30%) Use the pumping lemma to show that the following languages are not context free.
   (a) $\{\mathtt{0}^n\mathtt{1}^n\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$
   (b) $\{x_1\#x_2\#\cdots\#x_k \mid k \geq 2,$ each $x_i \in \{\mathtt{a},\mathtt{b}\}^*,$ and for some $i \neq j, x_i = x_j\}$
6. (Further studies: No marks) Let C be a context-free language and R be a regular language. Show that $C \cap R$ is context free. (Hint: Consider running a PDA and a DFA *in parallel*.)
7. (Further studies: No marks) Use the above result to show that the language

   $$\{w \mid w \in \{\mathtt{a},\mathtt{b},\mathtt{c}\}^* \text{ and contains equal numbers of } \mathtt{a}\text{'s, } \mathtt{b}\text{'s, and } \mathtt{c}\text{'s}\}$$

   is not context free.
8. (Further studies: No marks) Show that $F = \{\mathtt{a}^i\mathtt{b}^j \mid i = kj$ for some positive integer $k\}$ is not context free.
9. (Further studies: No marks) Study last year's homework, in particular, Ogden's Lemma.

[1] Sometimes, we may not specify the start variable in a CFG (just like the CFG in this question). In such a case, the *default* start variable is set to be the variable on the left-hand side of the first rule.