CS5314 Randomized Algorithms

Homework 5 – Optional
Due: January 05, 2012 (before class)

1. Let S be a set of elements. Let C be a collection of subsets of S, each containing k elements, where $k \geq 2$.

   Show that if $|C| \leq 4^{k-1} - 1$, we can color the elements of S with 4 colors such that no S_i is monochromatic (monochromatic means all elements have the same color).

   Hint: If we color the elements in S randomly, what is the probability that a particular S_i is monochromatic? What is the probability that some subset in C is monochromatic?

2. A tournament is a graph with exactly one directed edge between each pair of vertices. That is, a tournament looks like a complete graph, except that each edge is directed.

   Show that there is a tournament T with n vertices such that T contains at least $n!/2^{n-1}$ Hamiltonian paths.

   Hint: If we direct the edges randomly in a complete graph, what is the probability that a particular permutation of the vertices forms a Hamiltonian path? What is the expected number of Hamiltonian paths in the graph?