CS5314 Randomized Algorithms

Homework 4
Due: December 29, 2011 (before class)

1. Consider the two-state Markov chain with the following transition matrix.

$$\mathbf{P} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

Find a simple expression for $\mathbf{P}^t_{0,0}$.

2. Suppose that we have a Markov chain with three states, 0, 1, and 2. For state 0, we have probability 0.1 to stay and 0.9 to go to state 1. When we are at state 1, the probability to go to state 2 is 0.3, and the probability to go back to state 0 is 0.7. We would stay for probability 0.2 while we are at state 2 and go back to state 1 with probability 0.8.

   (a) Argue that the Markov chain is aperiodic and irreducible.

   (b) Find the stationary probability.

3. Consider the Markov chain in the following figure. Suppose that $n \geq 4$. Find the expected number of moves to reach n starting from position i, when (a) $i = n-1$, (b) $i = n-2$, (c) $i = n-3$, and (d) $i = n-4$. (Express the answers exactly in terms of n.)

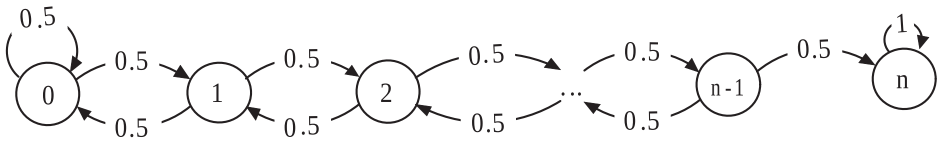

Figure 1: The Markov chain for Question 3.

4. Let L_n be an undirected graph with n nodes forming a line.

   (a) Argue that the cover time of L_n is $O(n^2)$.

   (b) Argue that the cover time of L_n is $\Omega(n^2)$.