Randomized Algorithm Tutorial 1

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Outline

- Hint for assignment 1
- Further studies
  - Mutually independent
  - Number of min-cuts in a graph
  - Proof of inclusion-exclusion principle

Question 1

- We draw cards uniformly at random without replacement from a standard deck of cards.
  - A: the event that the first two cards are red
  - B: the event that the third card is black
- We want to compute $\Pr(A)$, $\Pr(B)$, $\Pr(B|A)$, $\Pr(A|B)$

Question 1

- Hint:
  List the cases

Question 2

- There are three special fair dice, having the following numbers on their faces:
  - Die A – 1, 2, 2, 2, 3, 3
  - Die B – 1, 1, 2, 2, 3, 3
  - Die C – 1, 1, 2, 2, 2, 3

Question 2

- (a) You are going to throw die *A* and die *C*. What is the probability that the outcome of die *A* is greater than the outcome of die *C* ?
- (b) You throw the three dice in an unknown order, and you get two 2's and one 3. What is the probability that the die with outcome 3 is die *C* ?

Question 2

- Hint (a)
  List all the cases

- Hint (b)
  The conditional probability

Question 3

- X and Y are independent geometric random variables, where X has parameter p and Y has parameter q
  - $\Pr(X=Y)$
  - $E[\max(X,Y)]$
  - $\Pr(\min(X,Y) = k)$
  - $E[X|X \leq Y]$

Question 3

- Hint(c)

  min(X,Y) is the random variable denoting the number of steps that you see the first head
- Hint(b)

  $\max(X,Y) = X + Y - \min(X,Y)$

  memory-less property might help

Question 4

- We draw cards uniformly at random with replacement form a deck of n cards 2n times.
  - What is the expected number of cards that are not chosen at all?
  - What is the expected number of cards that are chosen exactly once?

Question 4

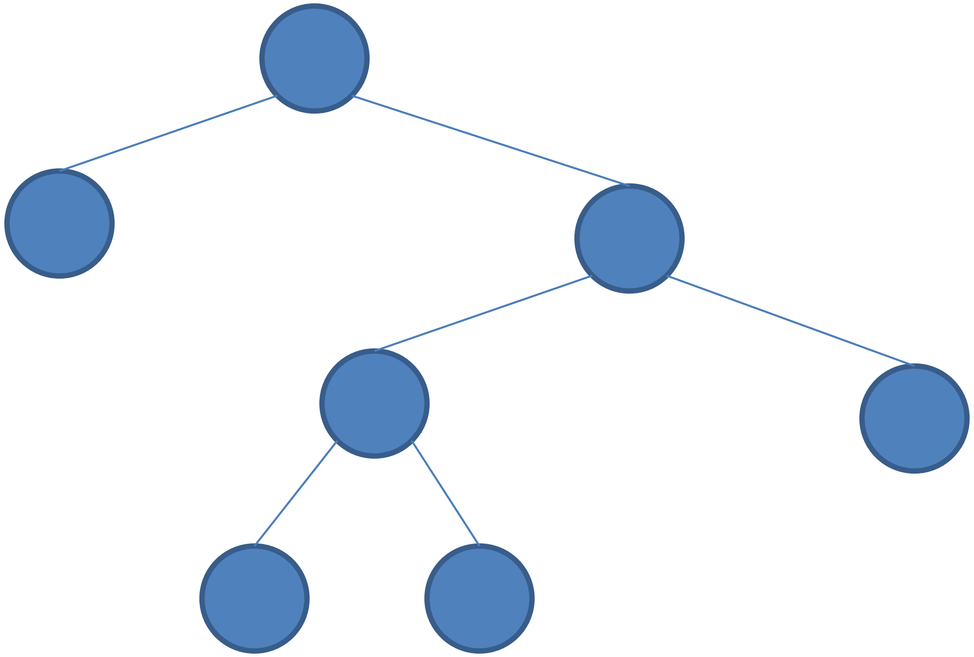
- Hint

  Use indicator random variable

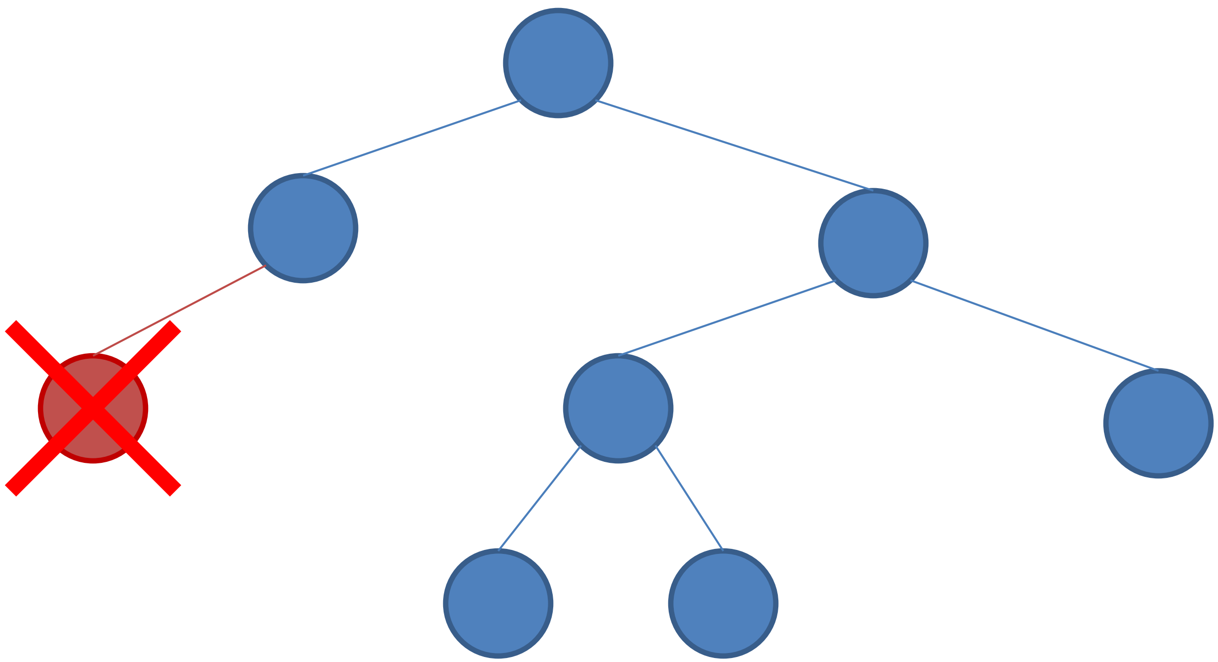
Question 5

- Recall the branching process problem. For each program P, before it finishes running, will create 2 new copies of itself with probability p, and 0 new copies with probability 1-p

Question 5



Question 5

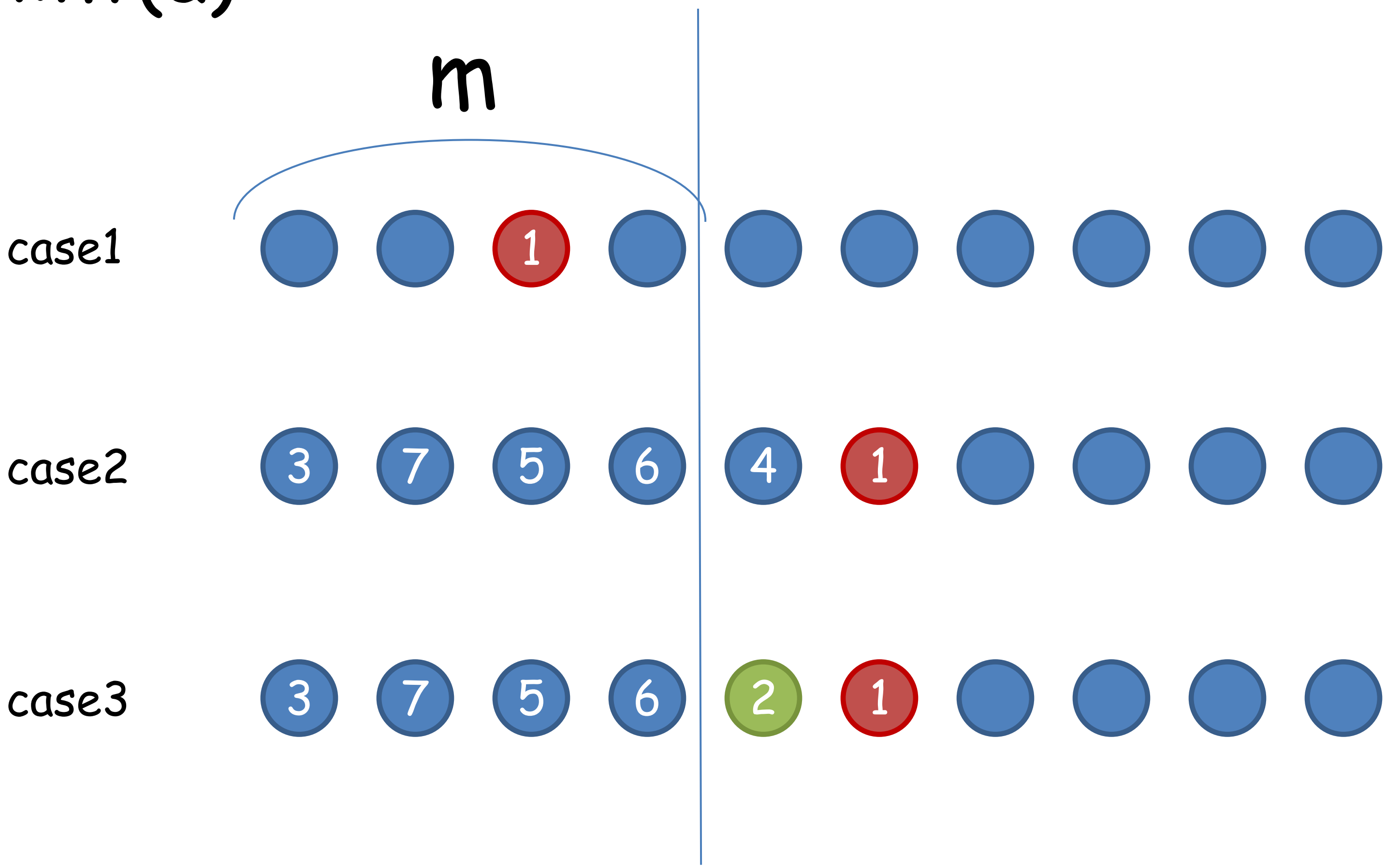


Question 6

- Choose the best one from n candidates!
- Algorithms:
  - Interview m candidates but reject them all
  - From the (m+1)th candidate, hire the first candidate who is better than all of the previous candidates you have interviews.
- What is the probability of hiring the best candidate?

Question 6

- Hint(a)



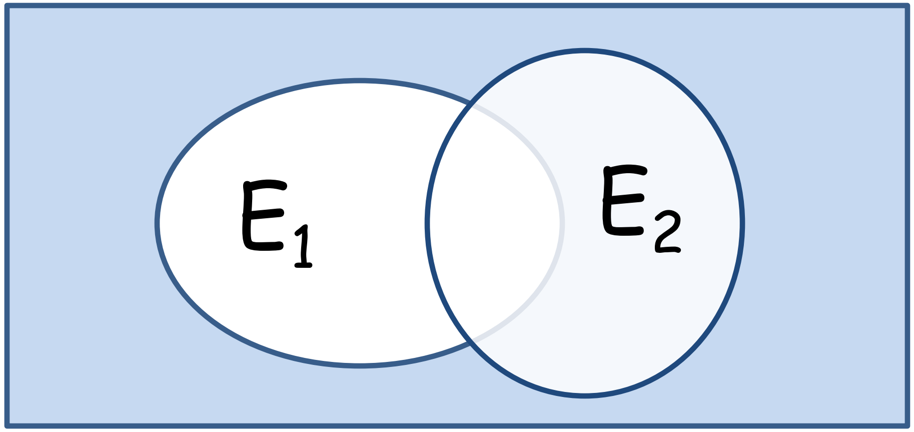
Further Studies

- If $E_1, E_2, \ldots, E_n$ are mutually independent, then the events $\overline{E_1}, \overline{E_2}, \ldots, \overline{E_n}$ are also mutually independent

Further Studies

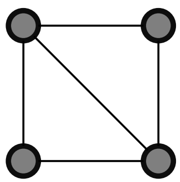
- If E_1 and E_2 are independent
  $\Pr(E_1 \cap E_2) = \Pr(E_1) * \Pr(E_2)$

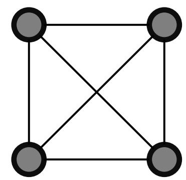
  $\Pr(\overline{E_1}) * \Pr(\overline{E_2}) = (1-\Pr(E_1)) * (1-\Pr(E_2))$
  $= 1 - \Pr(E_1) - \Pr(E_2) + \Pr(E_1 \cap E_2)$ (why?)
  $= \Pr(\overline{E_1} \cap \overline{E_2})$

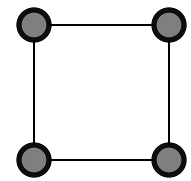


Further Studies

- # of min-cut







Size of min-cut = 2
min-cut = 2

Size of min-cut = 3
min-cut = 4

Size of min-cut = 2
min-cut = 6

- Can we have more min-cuts?

Inclusion-Exclusion Principle

- For any two events E,

  $\Pr(E \cup F) = \Pr(E) + \Pr(F) - \Pr(E \cap F)$

  $\Pr(E \cup F \cup G) = \Pr((E \cup F) \cup G)$

  $= \Pr(E \cup F) + \Pr(G) - \Pr((E \cup F) \cap G)$

Inclusion-Exclusion Principle

- $Pr(E \cup F \cup G) = Pr((E \cup F) \cup G)$

  $= Pr(E \cup F) + Pr(G) - Pr((E \cup F) \cap G)$

  $= Pr(E \cup F) + Pr(G) - Pr((E \cap G) \cup (F \cap G))$

  $= Pr(E \cup F) + Pr(G) - (Pr(E \cap G) + Pr (F \cap G) - \underline{Pr((E \cap G) \cap (F \cap G))})$

  $= Pr(E \cup F) + Pr(G) - (Pr(E \cap G) + Pr (F \cap G) - \underline{Pr(E \cap F \cap G)})$

  $= \ldots$