CS5314 Randomized Algorithms

Homework 5
Due: 2:10 pm, January 4, 2011 (to Jenny at Rm 734, General Building II)

1. (40%) Consider the Markov chain in the following figure. Suppose that $n \geq 4$. Find the expected number of moves to reach n starting from position i, when (a) $i = n-1$, (b) $i = n-2$, (c) $i = n-3$, and (d) $i = n-4$. (Express the answers exactly in terms of n.)

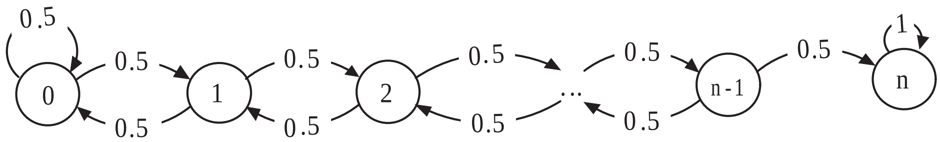

Figure 1: The Markov chain for Question 1.

2. Consider the Markov chain in the following figure.

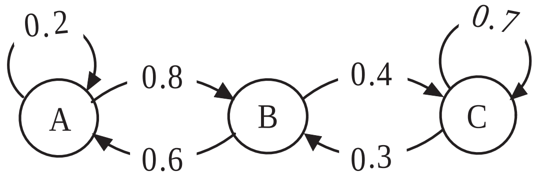

Figure 2: The Markov chain of Question 2.

   (a) (20%) Argue that the Markov chain is aperiodic and irreducible.
   (b) (20%) Find the stationary probability.
   (c) (20%) Find the probability of being in state 0 after 16 steps if the initial state is chosen uniformly at random from the 3 states.

3. (Just for fun: No marks) In the lecture, we have studied the gambler's ruin problem in the case where the game is fair. Consider the case where the game is not fair; in particular, the probability of losing a dollar each game is $2/3$ and the probability of winning a dollar each game is $1/3$.

   Suppose that you start with i dollars and finish either when you reach n or lose it all. Let W_t be the amount you have gained after t rounds of play.

   (a) Show that $E[2^{W_t}] = E[2^{W_{t+1}}]$.
   (b) Find the probability that you are winning.