# CS5314 Randomized Algorithms

Homework 5

Due: 2:10 pm, January 7, 2010 (before class)

1. (40%) Consider the Markov chain in the following figure. Suppose that $n \geq 4$. Find the expected number of moves to reach $n$ starting from position $i$, when (a) $i = n - 1$, (b) $i = n - 2$, (c) $i = n - 3$, and (d) $i = n - 4$. (Express the answers exactly in terms of $n$.)

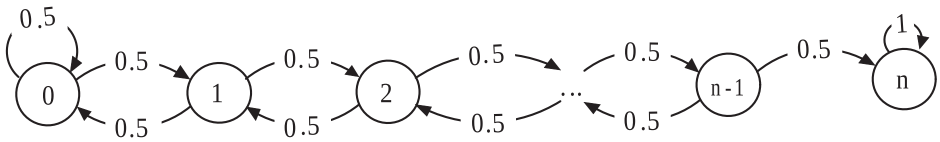


Figure 1: The Markov chain for Question 1.

2. Consider the Markov chain in the following figure.

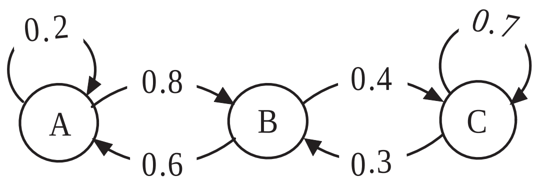


Figure 2: The Markov chain of Question 2.

   (a) (20%) Argue that the Markov chain is aperiodic and irreducible.
   (b) (20%) Find the stationary probability.
   (c) (20%) Find the probability of being in state 0 after 16 steps if the initial state is chosen uniformly at random from the 3 states.

3. (Just for fun: No marks) In the lecture, we have studied the gambler's ruin problem in the case where the game is fair. Consider the case where the game is not fair; in particular, the probability of losing a dollar each game is 2/3 and the probability of winning a dollar each game is 1/3.

   Suppose that you start with $i$ dollars and finish either w hen you reach $n$ or lose it all. Let $W_t$ be the amount you have gained after $t$ rounds of play.

   (a) Show that $E[2^{W_t}] = E[2^{W_{t+1}}]$.
   (b) Find the probability that you are winning.