CS5314 Randomized Algorithms

Homework 5
Due: 1:20 pm, Jan 6, 2009 (before class)

1. (100%) Suppose we have three states, 0, 1, and 2. For state 0, we have probability 0.4 to stay and 0.6 to go to state 1. When we are at state 1, the probability to go to state 2 is 0.3 and the probability to go back to state 0 is 0.7. We would stay for probability 0.8 while we are at state 2 and go back to state 1 with probability 0.2. This problem can be modeled by the following Markov chain.

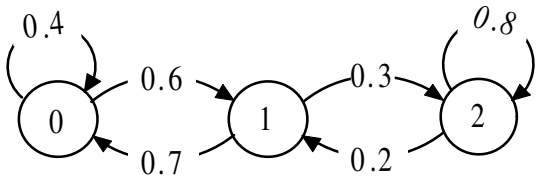

Figure 1: The modeling Markov chain of this question.

   (a) Argue that the Markov chain is aperiodic and irreducible.

   (b) Find the stationary probability.

2. (Bonus: 10%) The *lollipop* graph on n vertices is a clique on $n/2$ vertices connected to a path on $n/2$ vertices, as shown in the following. The node u is a part of both the clique and the path. Let v denote the other end of the path.

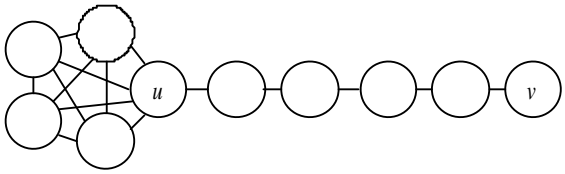

Figure 2: A lollipop graph.

   (a) Show that the expected covering time of a random walk starting at v is $\Theta(n^2)$.

   (b) Show that the expected covering time of a random walk starting at u is $\Theta(n^3)$.