CS5314 Randomized Algorithms

Homework 5
Due: 3:20 pm, June 12, 2008 (before class)

1. (100%) Suppose we have three states, 0, 1, and 2. For state 0, we have probability 0.1 to stay and 0.9 to go to state 1. When we are at state 1, the probability to go to state 2 is 0.3 and the probability to go back to state 0 is 0.7. We would stay for probability 0.2 while we are at state 2 and go back to state 1 with probability 0.8. This problem can be modeled by the following Markov chain.

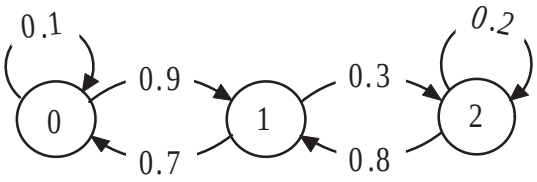

Figure 1: The modeling Markov chain of this question.

   (a) Argue that the Markov chain is aperiodic and irreducible.

   (b) Find the stationary probability.

2. (Bonus: 10%) We define a threshold queue with parameter T as follows: When the number of job is less than T, then the number of jobs decreases by 1 with probability 0.4 and increases by 1 with probability 0.6 at each step(time). However, when the number of jobs is larger than T, the number of jobs increases by 1 with probability 0.4 and decreases by 1 with probability 0.6 at each step(time).

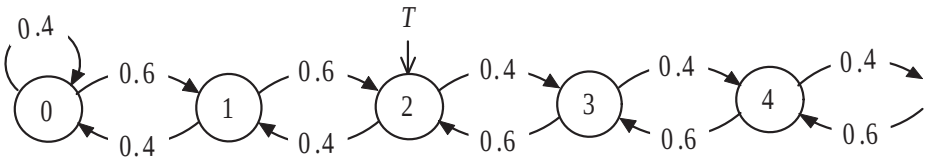

Figure 2: An example figure when $T = 2$.

   (a) Argue that the Markov chain is aperiodic and irreducible.

   (b) Compute the limiting probability distribution as a function of T, for arbitrary threshold T.

   (c) Compute the mean number of jobs in a threshold queue as a function of T.

   (d) What happens when $T = 0$? Does this answer make sense?