CS5314 Randomized Algorithms

Homework 5
Due: June 21, 2007 (before class)

1. Consider the two-state Markov chain with the following transition matrix.

$$\mathbf{P} = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix}$$

Find a simple expression for $P^t_{0,0}$.

2. (Further studies: No marks) We have considered the gambler's ruin problem in the case where the game is fair. Consider the case where the game is not fair; instead, the probability of losing a dollar each game is 2/3 and the probability of winning a dollar each game is 1/3. Suppose that you start with i dollars and finish either when you reach n or lose it all. Let W_t be the amount you have gained after t rounds of play.

   (a) Show that $E[2^{W_t}] = E[2^{W_{t+1}}]$.

   (b) Find the probability that you are winning.

3. (Further studies: No marks) The *lollipop* graph on n vertices is a clique on $n/2$ vertices connected to a path on $n/2$ vertices, as shown in Figure 1. The node u is a part of both the clique and the path. Let v denote the other end of the path.

   (a) Show that the expected covering time of a random walk starting at v is $\Theta(n^2)$.

   (b) Show that the expected covering time of a random walk starting at u is $\Theta(n^3)$.

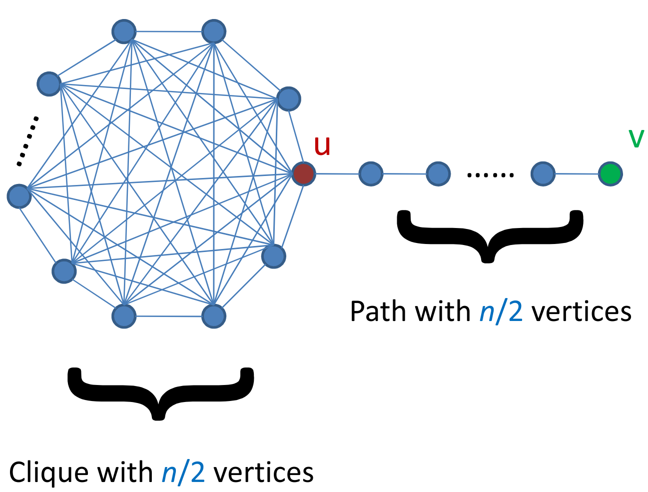

Figure 1: A lollipop graph