Theory of Computation Tutorial VI

Speaker: Yu-Han Lyu December 26, 2006

Theory Matters

- http://theorymatters.org
- Recent TCS breakthroughs
 - Boosting
 - Random Graph
 - Quantum Computation
 - Hashing
 - Interactive Proof
 - Probabilistic Checkable Proof
 - Smoothed Analysis

Theory Research

- 2006年9月28日,微軟亞洲研究院理論研究 組正式成立,中科院外籍院士姚期智擔任 研究組首席顧問
- "Viewpoint: the real reason why software engineers need math", Vol 44. No 10 CACM
- "Why computer science students need math", Vol 34, Issue 4, ACM SIGCSE

Computer Science

- 美國大學新鮮人對電腦科系敬而遠之
 - <u>http://taiwan.cnet.com/news/software/0,2000064574,2</u> 0098521,00.htm
- 專訪蓋茲:爲何電腦科系不再吃香?
 - <u>http://taiwan.cnet.com/news/special/0,2000064597,20</u> 088003,00.htm
- 美電腦科系不受學生青睞
 - http://taiwan.cnet.com/news/hardware/0,2000064553, 20091661,00.htm
- Impact factor
 - SIAM Journal on Computing
 - Journal of Complexity

Moderately Exponential

- m(n) is of moderately exponential growth if $\forall_{k>0} m(n) = \omega(n^k)$ and $\forall_{\varepsilon>0} m(n) = o((1 + \varepsilon)^n)$
- Example
 - n^{log n}, (log n)!
- If some NPC problem has moderately exponential lower bound, NP \neq P

NP-hard

- Given a language L, if every NP language can polynomial time reduction to L, then L is called NP-hard
- If L is NP and NP-hard, then L is NPcomplete.
- TM_{HALT} is NP-hard
 - Transform 3-SAT's input to a TM M, such that M tries all possible truth assignment.
 - If M finds one, accept. Otherwise, loop.

Closure Property

- Concatenation
 - P, NP
- Union
 - P, NP
- Star
 - -P, NP
- Intersection
 - P, NP
- Complement
 - P, NP?

$\mathsf{HAM}\text{-}\mathsf{CIRCUIT} \geqq_\mathsf{m} \mathsf{HAM}\text{-}\mathsf{PATH}$

- For any $(u,v) \in E$
 - Test whether exists a Ham-Path (u,v)
 - If one of $O(n^2)$ edge is accepted, accept.
- Is it a polynomial time many-one reduction?
- Why?

- BB is not computable function
- Suppose f is any computable function
- We combine 3 TM
 - Write n 1s with n state
 - Double 1s on the tape with c_1 state
 - Write f(2n) on the tape with c_2 state
- $BB(2n) > BB(n+c_1+c_2) \ge f(2n)$
- A_{TM}?

- $AMBIG_{CFG}$ is undecidable
- If P has a match $t_{i1}t_{i2}...t_{il}=b_{i1}b_{i2}...b_{il}$
 - $-t_{i1}t_{i2}\ldots t_{il}a_{il}\ldots a_{i2}a_{il}$ has two derivation, one from T and one from B
- If CFG is ambiguous
 - s=wa_{il}…a_{i2}a_{i1}, where w contains only symbols from P, and s is ambiguous
 - $-s = t_{i1}t_{i2}...t_{il}a_{il}...a_{i2}a_{i1}$
 - $-s=b_{i1}b_{i2}...b_{il}a_{il}...a_{i2}a_{i1}$

- A_{2DFA} is decidable
 - A 2DFA which has s states, on the input x will has at most $s(|x|+2)^2$ possible configurations
- E_{2DFA} is undecidable
 - $-E_{2DFA}$ can decide E_{TM}
 - Construct a 2DFA can accept any accepting computation history for TM M.
 - Start configuration and accept configuration
 - Legally follow configuration

- Let $J = \{ w \mid \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \notin A_{TM} \}$. Show that neither J nor complement of J is Turing-recognizable.
- $A'_{TM} \leq_m J$ - f(x) = 1x
- $A'_{TM} \leq_m J$'s complement -f(x) = 0x

- Rice Theorem
- Refer to textbook

Homework 5

- Due
 - 3:20 pm, January 5, 2007 (before class)
- Easiest : Problem 1
- Normal : Problem 2, 4
- Harder : Problem 3
- Hardest : Problem 5

• Show that HITSET is NP-complete.

- Let U = {<M, x, #^t> | TM M accepts input x within t steps on at least one branch}.
 Show that U is NP-complete.
- You can not prove by reducing any NPcomplete problem to U.
- By NP's definition!!

- Call a regular expression star-free if it does not contain any star operations.
- Let EQ_{SF-REX} = { <R, S> | R and S are equivalent star-free regular expressions}. Show that EQ_{SF-REX} is in coNP.
- Verifier is easier
- NP algorithm is also easy

- You are given $Q = \{q_0, q_1, ..., q_l\}$ and a collection $\{(s_1, r_1), ..., (s_k, r_k)\}$. s_i is string, r_i is state. Determine whether a DFA M exists where $\delta(q_0, s_i) = r_i$ for each i.
- 3-SAT

- Minimize DFA's algorithm
- Prove it is correct
- Prove it is minimum
 - Myhill-Nerode Theorem
- Prove it is in polynomial time.

HW5 – Self Learning

- Let f: N→N be any function where f(n) = o(n log n). Show that TIME(f(n)) contains only the regular languages.
- 1-tape TM
- HW3 Problem 1
- Myhill-Nerode Theorem
- "On the structure of one-tape nondeterministic Turing machine time hierarchy", Kojiro Kobayashi, Theoretical Computer Science Volume 40, Issue 2-3 (November 1985).