Theory of Computation Tutorial III

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Minimum Pumping Length

- 0001*
  - 4, because 000 can’t be pumped
- 0001
  - ??

Stochastic Context Free Grammar

- Each production is augmented with a probability
- Like Hidden Markov Model
  - Learning
- Application
  - Natural language processing
  - RNA

Context Sensitive Grammar

- Context Free Grammar
  - Left-hand side must be non-terminal
- $aA \rightarrow aB$
  - lhs length is smaller than or equal to rhs
- Example: $\{a^n b^n c^n | n \geq 1\}$
  - $S \rightarrow aSBc|abc$
  - $cB \rightarrow Bc$
  - $bB \rightarrow bb$

Linear Bounded Automata

- Like Turing machine, but the tape length is not infinite
- Tape length is linear to the input length
- LBA and DLBA are not equivalent
  - DLBA : deterministic subset of LBA
- LBA = CSG
- Chapter 5

Unrestricted Grammar

- $aA \rightarrow aB$
  - No length limitation
- Unrestricted Grammar = Recognizable

Chomsky Hierarchy

Type	Grammar	Language	Automaton	
Type-3	Linear	Regular	NFA	NFA=DFA
Type-2	Context Free	Context Free	PDA	PDA$\neq$DPDA
Type-1	Context Sensitive	Context Sensitive	LBA	LBA$\neq$DLBA
		Recursive	Decider	
Type-0	Unrestricted	Recursive Enumerable	Turing Machine	TM=DTM

Closure properties

Closed?	$\cup$	$\cap$	complement	concatenation	star
Regular	Y	Y	Y	Y	Y
Context Free	Y	N	N	Y	Y
Context Sensitive	Y	Y	Y	Y	Y
Recursive	Y	Y	Y	Y	Y
Recursive Enumerable	Y	Y	N	Y	Y

Decidability

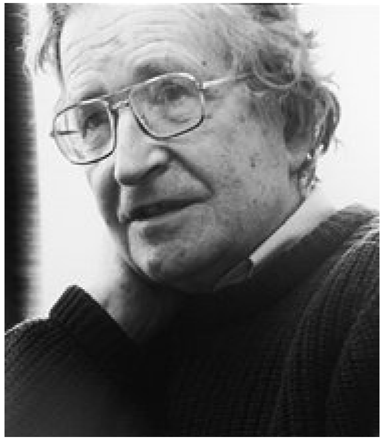
Decidable?	Accept	Empty	Equal
Regular	Y	Y	Y
Context Free	Y	Y	N
Context Sensitive	Y	N	N
Recursive Enumerable	N	N	N

Question

- Let M_1 be an NTM. Suppose that M_1 is a decider for language L. If we exchange the accept state and reject state and get another NTM M_2, is it a decider for L's complement?
  - No
  - If M_2 accepts x, it means that there exists one path to M_1's reject state
  - But L's complement means that there is no path to accept state

Chomsky

- Institute Professor Emeritus of linguistics at the Massachusetts Institute of Technology



Complexity

- Computational Complexity
  - Time, Space, ….
- Information Entropy
  - Complexity in data
- Descriptive Complexity
  - Complexity to specify the data
  - Kolmogorov Complexity (6.4)
  - Algorithmic Information Theory
  - An Introduction to Kolmogorov Complexity and Its Applications, by Ming Li and Paul Vitanyi

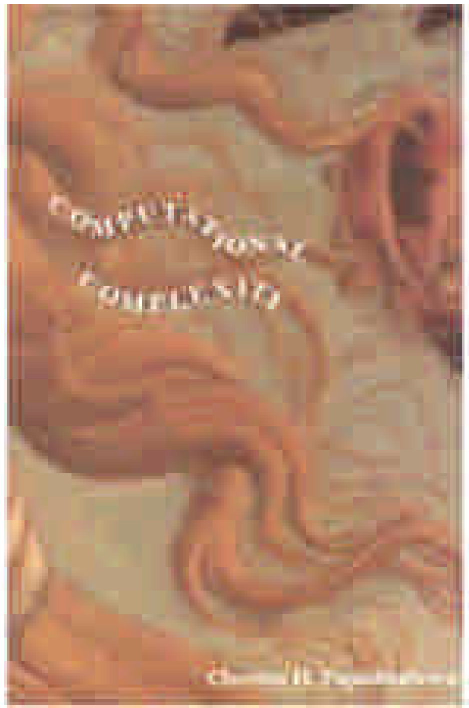
Logic & Recursion theory

- 1931 – Godel Incompleteness
- 1933 – Godel developed the ideas of computability and recursive functions
- Lambda calculus by Church and Kleene
- 1936 – Turing Machine by Turing, Formulation 1 by Post

Syntactic/ Logic	Semantic/ Automata
Primitive Recursive	
Total Recursive	Recursive
Partial Recursive	Recursive Enumerable

Reference 1

- Computational Complexity
- Christos H. Papadimitriou
- 1994~



Reference 2

- Complexity Theory: A Modern Approach
  - Sanjeev Arora and Boaz Barak
  - http://www.cs.princeton.edu/theory/complexity
  - First part covers the Papadimitriou's book
- Computational Complexity: A Conceptual Perspective
  - Oded Goldreich
  - http://www.wisdom.weizmann.ac.il/~oded/cc-book.html
- Not published

Reference 3

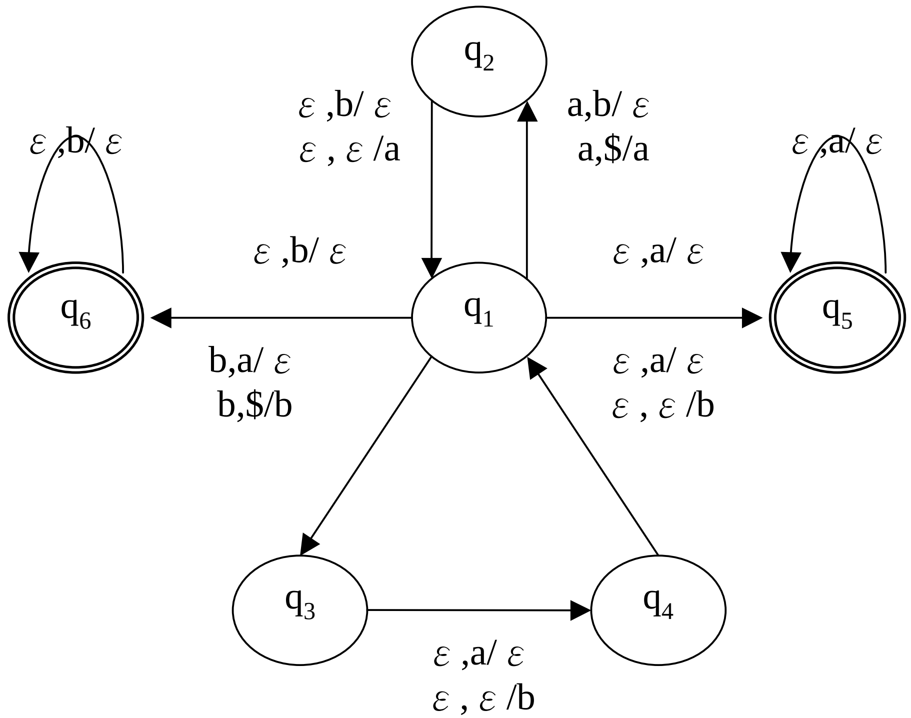
- Complexity Zoo
  - http://qwiki.caltech.edu/wiki/Complexity_Zoo
- Wikipedia
  - http://en.wikipedia.org/wiki/Main_Page

Problem 1

- Every tree with height k has at most 2^k-1 internal nodes.
- 2^b derivation steps $\rightarrow$ height > b
- At least one variable occurs twice
- Pumping lemma..

Problem 2

- If we read a ‘a’
  - Eliminate 2b
  - Push 2a
- If we read a ‘b’
  - Eliminate 3a
  - Push 3b
- After read input
  - If stack isn’t empty then accept



Problem 3

- $C = \{xy \mid x, y \in \{0,1\}^* \text{ and } |x|=|y|, x \neq y\}$
- First half and second half are different in some position
- $S \rightarrow AB|BA$
- $A \rightarrow XAX|0$
- $B \rightarrow XBX|1$
- $X \rightarrow 0|1$

Problem 4

- $A = \{ww^R \mid w,t \in \{0,1\}^* \text{ and } |w| = |t|\}$
- $S=0^{2p}0^{p}1^{p}0^{2p}$
- If v and y are all 0 or all 1
  - Impossible
- If v is 0 and y is 1
  - Impossible
- If v is 1 and y is 0
  - Impossible

Problem 5

- An internal node is marked if it has at least two children, and both of them contain a marked leaf as a descent
- Prove by induction that if every path of T contains at most i marked node, then T has at most b^i marked leaves
- $K=b^{|V|+1}$

Problem 6

- $z=a^p b^p c^{p+p!}=uvxyz$
- $s_1=uv^2xy^2z$, $\#_b(s) \leqq 2p<p+p! \leqq \#_c(s)$
  - $\#a(s)=\#b(s)$
  - $v=a^t$ and $x=b^t$ for some t
- Let $n=p!/t + 1$
  - $s_1'=uv^nxy^nz=a^{p+p!}b^{p+p!}c^{p+p!}$