Theory of Computation Tutorial II

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Closed operations for CFL

- Union
- Concatenation
- Star
- Reverse
- Complement is not closed
- Intersection with regular language
- Difference with regular language
- Quotient with regular language

Right Linear Grammar

- $A \to wB$
- $A \to w$
- Right Linear Grammar = Regular Language
- Left linear ?

Deterministic PDA

- NPDA $\neq$ DPDA
- DPDA $\rightarrow$ Unambiguous
- LR(k)
- How about LL, LALR, SLR??

Assignment 1

- Problem 1
  - Easiest
- Problem 2
  - Special cases: 0, 1, ε
- Problem 7
  - Answer is in textbook

Problem 3

- Assume there exists a pumping length p
- We find a string $s=w^n$, $n>p$, n is prime
- No matter how we divide s=xyz
  - $xy^{|n+1|}z = n+n|y| = n(1+|y|)$, which is not prime

Pumping lemma

- If L is regular language, then there exists a pumping length p, such that for any string s, |s|>p and s$\in$ L, we can divide s into three parts………..
- L is regular $\rightarrow$

$\{\exists_p \forall_s (|s| \geq p \text{ and } s \in L) \rightarrow$

$[\exists_{s=xyz} |y|>0 \text{ and } |xy| \leq p \text{ and } \forall_{i \geq 0}\, xy^iz \in L]\}$

To prove L is non-regular

- L is regular $\rightarrow$

  $\{\exists_p \forall_s (|s| \geq p \text{ and } s \in L) \rightarrow$

  $[\exists_{s=xyz} |y|>0 \text{ and } |xy| \leq p \text{ and } \forall_{i \geq 0}\ xy^iz \in L]\}$
- To prove L is non-regular, we show that: No matter what p is, we can find a string s, $|s| \geq p$, $s \in L$. But then, no matter how we divide s into xyz, at least one condition don't hold.

To prove L is consistent with pumping lemma

- L is regular $\rightarrow$

  $\{\exists_p \forall_s$ ($|s| \geq p$ and $s \in L$) $\rightarrow$

  $[\exists_{s=xyz}$ $|y|>0$ and $|xy| \leq p$ and $\forall_{i \geq 0}$ $xy^iz \in L]\}$

- We can find a p, such that **for any** string s, $|s| \geq p$ and $s \in L$, we can divide it into three parts…

Problem 4

- $F=\{a^i b^j c^k \mid i, j, k \geq 0$, if $i=1$, then $j=k\}$
- For any string s, $s \in F$
  - if s is the form $a^i b^j c^k$ $(i \neq 2)$
    - $x=\varepsilon$, y= first character, z = remainder ...
  - If s is the form $aab^j c^k$ (i=2)
    - $x=\varepsilon$, y=aa, z = remainder ...
- What is the pumping length?

Problem 5

- Alternately run in A and B
- Keep the information
  - A and B states $\rightarrow$ A x B
  - The next input will run in A or B $\rightarrow$ {Odd, Even}

Problem 6

- The same as problem 5
- In state (a, b) we can
  - Run on A
  - Run on B

Problem 8

- Non-deterministically guess a final state
- Bidirectional process
  - Forward: Run A
  - Backward: Run A^R
- If end in the same state then accept

Assignment 2

- Due: 2:10 pm, October 31, 2006 (before class)
  - Late submission will not be marked

Problem 1

- Proof of pumping lemma

Problem 2

- $A = \{w \mid 2\#a(w) \neq 3\#b(w), w \in \{a,b\}^*\}$
- When read a
  - Push a
  - Eliminate b
- When read b
  - Push b
  - Eliminate a

Problem 3

- Let $C = \{xy \mid x, y \in \{0,1\}^*$ and $|x|=|y|, x \neq y\}$. Show that C is a context-free language.
- At least one position is not equal
- The i^{th} position of the first half is not equal to i^{th} position of the second half.
- 4 variables, don't think too difficult

Problem 4

- Let $A = \{wtw^R \mid w,t \in \{0,1\}^* \text{ and } |w| = |t|\}$. Prove that A is not a context-free language.
- Pumping lemma

Problem 5

- Let L be a context-free language. Then there is a constant p such that for any string z in L with at least p characters, we can mark any p or more positions in z to be distinguished, and then z can be written as z = uvwxy, satisfying the following conditions:
  - (i) vwx has at most p distinguished positions.
  - (ii) vx has at least one distinguished position.
  - (iii) For all $i \geqq 0$, uv^iwx^iy is in L.
- Formal proof for all conditions

Problem 6

- Apply Ogden’s lemma and show that the language $L = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$ is inherently ambiguous.