Theory of Computation Tutorial I

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Closed operations

- Union
- Concatenation
- Star
- Complement: $L' = \sum^* - L$
  - Final state $\rightarrow$ non-final state
  - Non-final state $\rightarrow$ final state
- Difference
  - $L\text{-}M = (L' \cup M)'$

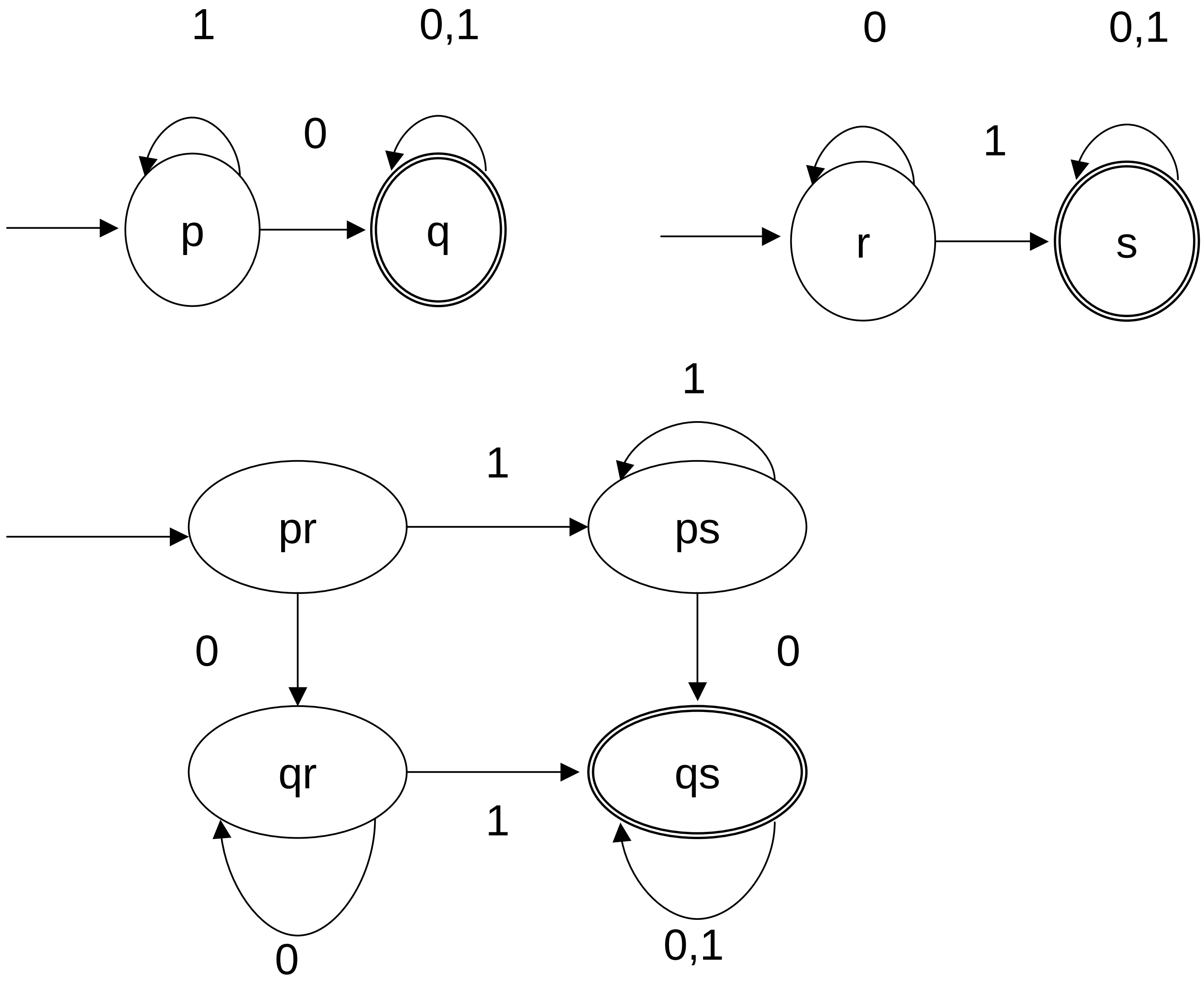
Intersection

- If A and B are regular languages, then so is $A \cap B$
- Proof
  - Regular language is closed under complement and union operations.
  - By DeMorgan's laws, we can use complement and union to construct intersection.

Another Proof

- Let two DFAs $D_A=(Q_A, \Sigma, \delta_A, q_A, F_A)$ and $D_B=$ $=(Q_B, \Sigma, \delta_B, q_B, F_B)$, $L(D_A)=A$, $L(D_B)=B$
- Parallel run two machines, if both accept then accept, otherwise reject.
- Formally, we construct DFA $D=(Q, \Sigma, \delta, q, F)$
  - $Q=Q_A \times Q_B$ (two tuple)
  - $F=F_A \times F_B$
  - Start state$=(q_A, q_B)$
  - $\delta((p,q),a)=(\delta_A(p,a), \delta_B(q,a))$
- Finally, we should prove $L(D) = L(A \cap B)$

Example



Reverse

- $A^R = \{w^R \mid w \in A\}$
- Reverse all the transitions
- Start state $\rightarrow$ final state
- Final state(s) $\rightarrow$ start state
- Closed

Quotient

- $A/B=\{w \mid wx \in A \text{ for some } x \in B\}$
- Run A's DFA
- Non-deterministically choose one state in A and guess x

Assignment 1

- Due: 3:20 pm, October 13, 2006 (before class)
  - Late submission will not be marked
- No cheating
  - Can exchange high-level idea
- Problems 1 ~ 3 are easy
- Problem 4
  - Use closed operation property to prove this language is not regular.

Problem 5

- Perfect shuffle of A and B language
  - {w | w=$a_1b_1\ldots a_kb_k$, where $a_1\ldots a_k \in$ A and $b_1\ldots b_k \in$ B, each a_i, $b_i \in \Sigma$ }
- Example
  - A={“abc”}
  - B={“def”}
  - “adbecf” $\in$ Perfect-shuffle(A,B)

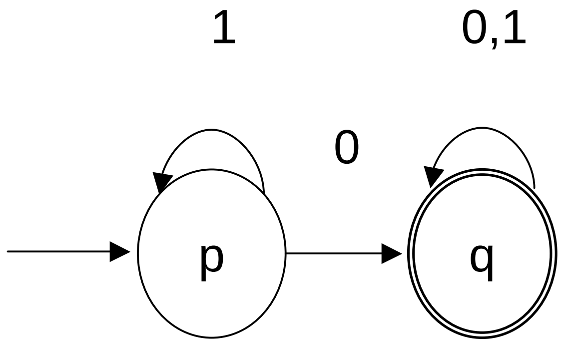
Idea

- When reading a character a, we should know
  - This character is in odd or even position
  - The current state in A and B
- Problem 6 is similar

Problem 7

- Answer is in the textbook
  - After understanding, write it down in you words, otherwise..
- x and y are distinguishable by L
  - Some string z exists whereby exactly one of the strings xz and yz is a member of L
- We say that X is pairwise distinguishable by L
  - Every two distinct strings in X are distinguishable by L.
- index of L
  - Maximum number of elements in any set of strings that is pairwise distinguishable by L

Example



- This language contains at least one zero
- “01” and “00” is indistinguishable
- “11” and “01” is distingushable by L, because “111” is not in L but “011” is in L (z=“1”)
- Index = 2

Myhill-Nerode Theorem

- L is regular **if and only** if it has finite index
- Application
  - Minimization DFA's state in unique (NFA??)
  - Proof for non-regular
- Example: L={x | x is palindrome, $x=x^R$}
  - $X=\{a^i \mid i \geqq 0\}$
  - a and aa are distingushable, by choosing z= ba
  - aa and aaa are distingushable, by choosing z= baa
  - index is infinite, so this language is not regular

Problem 8

- $A_{1/2} = \{ x \mid \text{for some } y, |x| = |y| \text{ and } xy \in A\}$
- No hint
- Harder problem: $A_{3/3} = \{ z \mid \text{for some } x, y, |x| = |y| = |z| \text{ and } xyz \in A\}$
  - Can your method extend to this problem?

Reference

- Introduction to Automata Theory, Languages, and Computation (3rd Edition), by John E. Hopcroft, Rajeev Motwani Jeffrey D. Ullman