CS2351 Data Structures

Homework 3

Due: May 9, 2011 (before class)

1. Consider the mathematical expression $12 \div 6 + 7 \times 5 \times 2 - 4$.

   Note that $\div$ and $\times$ have higher priority than $+$ and $-$, as usual. For instance, $1 + 2 \times 3$ means $1 + (2 \times 3)$ instead of $(1 + 2) \times 3$. Also, all these operators are *left-to-right* operator. For instance, $1 + 2 + 3$ means $(1 + 2) + 3$ instead of $1 + (2 + 3)$.

   (a) Build the expression tree of the expression.

   (b) Write down the prefix notation and the postfix notation of the expression.

   (c) Evaluate postfix notation with a stack, and show the key steps.

2. Recall that a stack is a *last-in-first-out* list that always inserts or removes an item from the end. These two operations are more popularly known as `push` and `pop`, respectively. By using a linked list or an array to represent a stack, each operation can be performed in $O(1)$ time.

   Your friend, Peter, wants to maintain a stack for storing numbers, but with an extra function called `find-min` which reports the value of the minimum item in the current stack. For instance, after `push 3`, `push 2`, `push 7`, and `push 1`, calling `find-min` at this point should return 1. If a `pop` operation is now followed, calling `find-min` again should return 2.

   Describe how to implement a stack so that each of the `push`, `pop`, and `find-min` operations can be performed in $O(1)$ time.

3. Let G be a directed graph and M be its adjacency matrix.

   (a) Let $M^2 = M \times M$ where the current $\times$ is matrix multiplication. Explain the physical meaning of the value of each entry in M^2. (Hint: Recall that $M[u, v] = 1$ if there is a directed edge from u to v, and $M[u, v] = 0$ otherwise. Now, what does the value in $M^2[u, v]$ mean?)

   (b) A *triangle* in G is defined as a sequence of vertices (u, v, w, u) such that $(u, v), (v, w)$, and (w, u) are directed edges found in G.

   Design an algorithm to check whether there is a triangle in G? Can you use only *one* matrix multiplication?