1. Consider the mathematical expression $12 \div 6 + 7 \times 5 \times 2 - 4$.

   Note that $\div$ and $\times$ have higher priority than $+$ and $-$, as usual. For instance, $1 + 2 \times 3$ means $1 + (2 \times 3)$ instead of $(1 + 2) \times 3$. Also, all these operators are left-to-right operator.

   For instance, $1 + 2 + 3$ means $(1 + 2) + 3$ instead of $1 + (2 + 3)$.

   (a) Build the expression tree of the expression.

   (b) Write down the prefix notation and the postfix notation of the expression.

   (c) Evaluate postfix notation with a stack, and show the key steps.

2. Recall that a stack is a last-in-first-out list that always inserts or removes an item from the end. These two operations are more popularly known as push and pop, respectively. By using a linked list or an array to represent a stack, each operation can be performed in $O(1)$ time.

   Your friend, Peter, wants to maintain a stack for storing numbers, but with an extra function called find-min which reports the value of the minimum item in the current stack. For instance, after push 3, push 2, push 7, and push 1, calling find-min at this point should return 1. If a pop operation is now followed, calling find-min again should return 2.

   Describe how to implement a stack so that each of the push, pop, and find-min operations can be performed in $O(1)$ time.

3. Let $G$ be a directed graph and $M$ be its adjacency matrix.

   (a) Let $M^2 = M \times M$ where the current $\times$ is matrix multiplication. Explain the physical meaning of the value of each entry in $M^2$. (Hint: Recall that $M[u, v] = 1$ if there is a directed edge from $u$ to $v$, and $M[u, v] = 0$ otherwise. Now, what does the value in $M^2[u, v]$ mean?)

   (b) A triangle in $G$ is defined as a sequence of vertices $(u, v, w, u)$ such that $(u, v), (v, w),$ and $(w, u)$ are directed edges found in $G$.

   Design an algorithm to check whether there is a triangle in $G$? Can you use only one matrix multiplication?