CS2351
Data Structures

Lecture 20:
Suffix Tree and Suffix Array

About this lecture

- So far, we have described data structure for searching numbers
- We now introduce two data structures for searching strings
  - Suffix Tree and Suffix Array

Text Indexing

String Matching problem:

Given a text T and a pattern P, how to locate all occurrences of P in T ?

- KMP algorithm can solve this in O(|T|+|P|) time ➔ optimal

- In some applications, T is very long, and given in advance, and we will search different patterns against it later
  - E.g., T= Human DNA, P = gene

Text Indexing

Text Indexing problem:

Suppose a text T is known.

Can we build a data structure for T, such that for any pattern P given later, we can find all occurrences of P in T quickly ?

- The data structure is called an index of T
- Target: search better than O(|T|+|P|) ??

Text Indexing

- Two main kinds of text indexes:

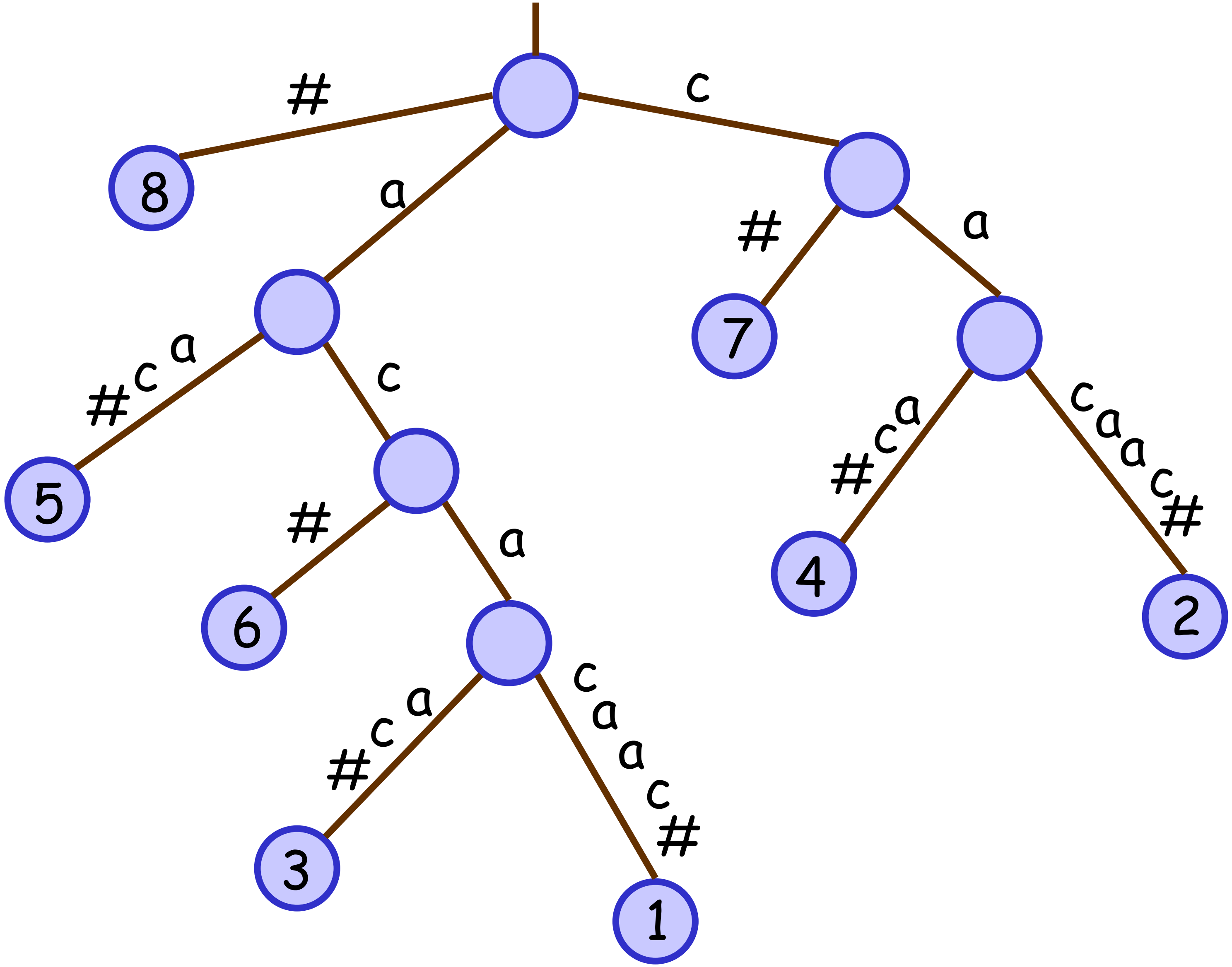
  Word-Based: (for texts formed by words)
  - Used by most text search engine
  - E.g., Inverted Files

  Full-Text: (for texts with no word boundaries)
  - Used in indexing DNA
  - E.g., Suffix Tree, Suffix Array

Suffix Tree

- Let T[1..n] be a text with n characters
  - we assume T[n] is a unique character
- For any j, T[j..n] is called a suffix of T

  ➔ T has exactly n suffixes
- Weiner (1973) and McCreight (1976) independently invented the suffix tree
  - a tree formed by putting all suffixes of T together



Suffix Tree of acacaac#

Definition of a Suffix Tree

- Suffix tree is an edge-labeled compact tree (no degree-1 nodes) with n leaves
  - each leaf ⇔ suffix
  - leaf label ⇔ starting pos of suffix
  - If we traverse from root to leaf k :

    edge labels along path ⇔ suffix T[k..n]
  - edge-label to each child starts with different character

Searching in a Suffix Tree

Theorem: If a pattern P occurs at position j in T, P is a prefix of T[j..n]

This suggests the searching algorithm below:

- Start from root of the suffix tree
- Traverse the suffix tree using P

➔ What we are doing is to match P with all suffixes of T at the same time

Searching in a Suffix Tree

Theorem: Pattern P occurs in T if and only if all chars of P are matched in the traversal of the searching algorithm

Questions:

1. How to locate the occurrences?
2. What is the searching time?

   $O(|P|+r)$ time, where r = #occurrences

Space Usage

- There are $O(n)$ nodes and $O(n)$ edges in the suffix tree

  → $O(n)$ space ?

- Each edge needs to store its label, which can contain $O(n)$ chars

  → In the worst-case, total $O(n^2)$ chars

- Can we reduce space usage?

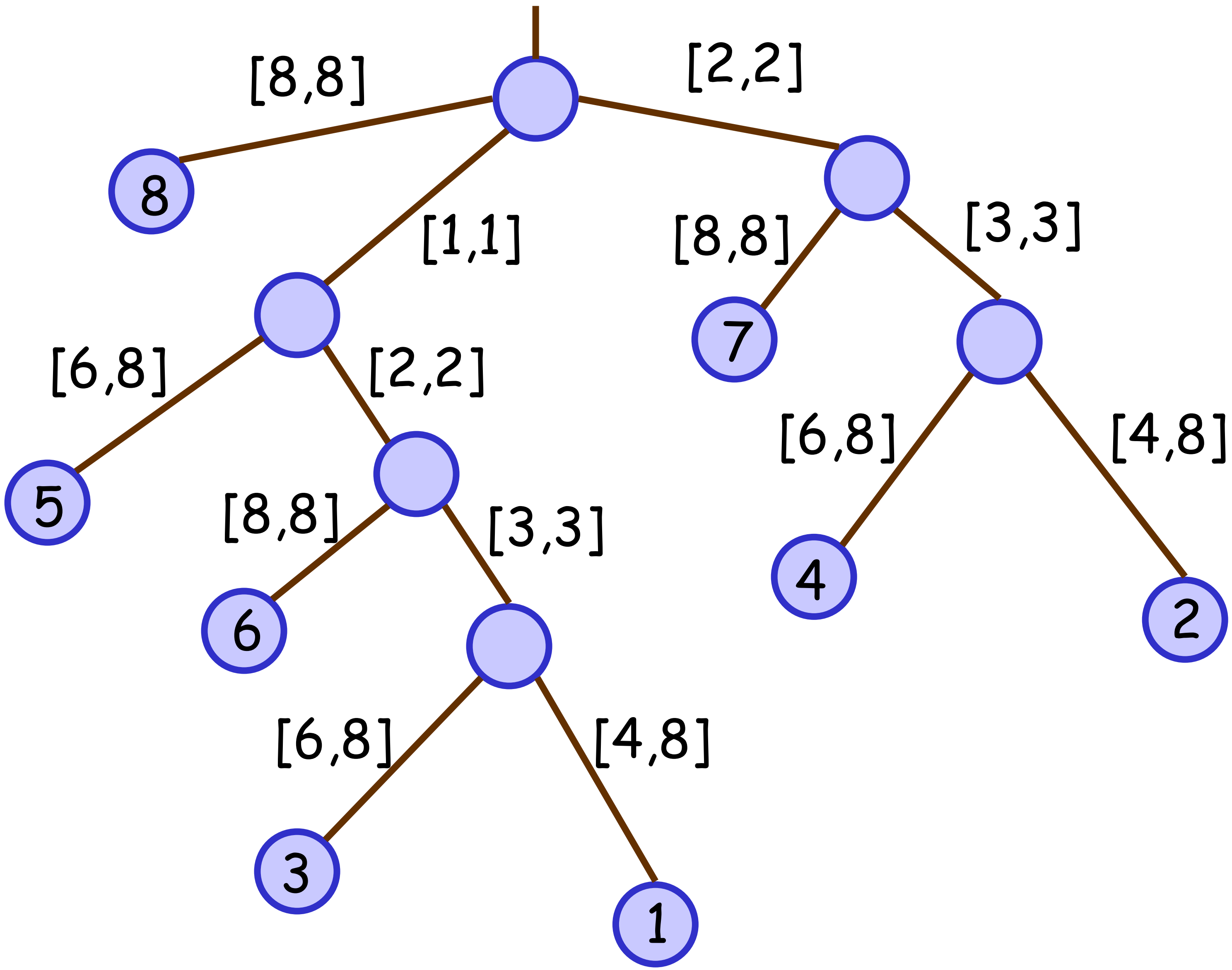
Space Usage

Observation: Each edge label must be equal to some substring of T

Clever Idea:

1. Store T, and
2. Replace each edge label by 2 integers, telling which substring it is equal to

➔ Total space: O(n)



Suffix Tree of acacaac#

Suffix Array

- Although suffix tree takes $O(n)$ space, the hidden constant is quite large
  ➔ around $40n$ to $60n$ bytes

- Manber and Myers (1990) simplified the suffix tree, and invented the suffix array
  - An array storing the suffixes of T in the "dictionary" order

Suffix Array

Suffix Array
of acacaac#

1	#
2	aac#
3	ac#
4	acaac#
5	acacaac#
6	c#
7	caac#
8	cacaac#

- The suffix array *SA* for *T* has *n* entries
- For any *j*, *SA*[*j*] stores the *j*th smallest suffix, based on alphabetical order
- Theorem: If *P* occurs in *T*, its occurrences correspond to consecutive region in *SA*

Suffix Array

Suffix Array of acacaac#

1	#
2	aac#
3	ac#
4	acaac#
5	acacaac#
6	c#
7	caac#
8	cacaac#

➔ Searching P takes $O(|P| \log n)$ time using binary search

Space:

We can represent each suffix by its starting position ➔ $O(n)$ space

In practice, around $14n$ bytes