CS2351
Data Structures

Lecture 1: Getting Started

About this lecture

- Study some sorting algorithms
  - Insertion Sort
  - Selection Sort
  - Merge Sort
- Show why these algorithms are correct
- Analyze the efficiency of the algorithms

The Sorting Problem

Input: A list of n numbers

Output: Arrange the numbers in increasing order

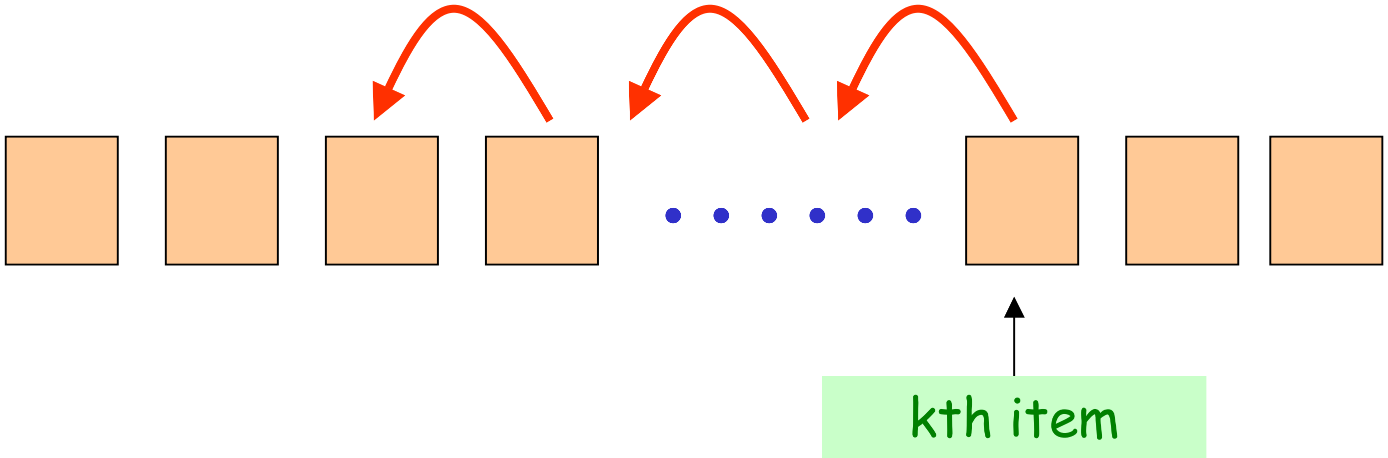
Remark: Sorting has many applications.

E.g., if the list is already sorted, we can search a number in the list faster

Insertion Sort

- Operates in n rounds
- At the k^{th} round,

Swap towards left side ;
Stop until seeing an item
with a smaller value.



Question: Why is this algorithm correct?

Selection Sort

- Operates in n rounds
- At the k^{th} round,
  - Find minimum item after $(k-1)^{th}$ position
  - Let's call this minimum item X
  - Insert X at k^{th} position in the list

Question: Why is this algorithm correct?

Divide and Conquer

- Divide a big problem into smaller problems
  ➔ solve smaller problems separately
  ➔ combine the results to solve original one
- This idea is called Divide-and-Conquer
- Smart idea to solve complex problems (why?)
- Can we apply this idea for sorting ?

Divide-and-Conquer for Sorting

- What is a smaller problem ?
  ➔ E.g., sorting fewer numbers
  ➔ Let's divide the list to two shorter lists
- Next, solve smaller problems (how?)
- Finally, combine the results
  ➔ "merging" two sorted lists into a single sorted list (how?)

Merge Sort

- The previous algorithm, using divide-and-conquer approach, is called Merge Sort
- The key steps are summarized as follows:

Step 1. Divide list to two halves, A and B

Step 2. Sort A using Merge Sort

Step 3. Sort B using Merge Sort

Step 4. Merge sorted lists of A and B

Question: Why is this algorithm correct?

Analyzing the Running Times

- Which of previous algorithms is the best?
- Compare their running time on a computer
  - But there are many kinds of computers !!!

Standard assumption: Our computer is a RAM (Random Access Machine), so that

- each arithmetic (such as $+, -, \times, \div$), memory access, and control (such as conditional jump, subroutine call, return) takes constant amount of time

Analyzing the Running Times

- Suppose that our algorithms are now described in terms of RAM operations

  ➔ we can count # of each operation used

  ➔ we can measure the running time !

- Running time is usually measured as a function of the input size
  - E.g., n in our sorting problem

Insertion Sort (Running Time)

Below is a pseudo-code for Insertion Sort :

```
Insertion-Sort(A)
1.    for j = 2 to length[A] {
1.1      Compare A_j with A_{j-1}, A_{j-2}, ...
         until getting A_x smaller than A_j ;
1.2      Insert A_j after A_x ;
      }
```

Note : Steps 1.1 and 1.2 can be described in terms of RAM operations. Can you do that ?

Insertion Sort (Running Time)

- Let $T(n)$ denote the running time of insertion sort, on an input of size n
- Suppose t_j denotes the number of comparisons in round j
- By combining terms, we have

$$T(n) = c_1(n-1) + c_2 \Sigma t_j$$

- The values of t_j are dependent on the input (not the input size)

Insertion Sort (Running Time)

- **Best Case:**
  The input list is sorted, so that all $t_j = 1$
  Then, $T(n) = c_1(n-1) + c_2(n-1)$
  $= Kn + c$ ➔ linear function of n
- **Worst Case:**
  The input list is sorted in decreasing order, so that all $t_j = j-1$
  Then, $T(n) = K_1n^2 + K_2n + K_3$
  ➔ quadratic function of n

Worst-Case Running Time

- In our course (and in most CS research), we concentrate on worst-case time
- Some reasons for this:
  1. Gives an upper bound of running time
  2. Worst case occurs fairly often

Remark: Some people also study average-case running time (they assume input is drawn according to some distribution)

Practice Implementation

- When we understand completely about an algorithm, it is easy to implement it using any programming language (such as C, C++, java)

- Can we write Insertion Sort in C ?

Merge Sort (Running Time)

The following is a partial pseudo-code for Merge Sort.

```
MERGE-SORT(A, p, r)
if p < r
   then q ← ⌊(p + r)/2⌋
        MERGE-SORT(A, p, q)
        MERGE-SORT(A, q + 1, r)
        MERGE(A, p, q, r)
```

The subroutine MERGE(A,p,q,r) is missing.

Can you complete it?

Hint: Create a temp array for merging

Merge Sort (Running Time)

- Let $T(n)$ denote the running time of merge sort, on an input of size n
- Suppose we know that Merge( ) of two lists of total size n runs in $c_1 n$ time
- Then, we can write $T(n)$ as:

  $T(n) = 2T(n/2) + c_1 n + c_2$ when $n > 1$

  $T(n) = c_3$ when $n = 1$
- Solving the recurrence, we have
- $T(n) = K_1\, n \log n + K_2\, n + K_3$

Which Algorithm is Faster?

- Unfortunately, we still cannot tell
  - since constants in running times are unknown
- But we do know that if n is VERY large, worst-case time of Merge Sort must be smaller than that of Insertion Sort
- Merge Sort is asymptotically faster than Insertion Sort

Practice Implementation

- Can we write Merge Sort in C ?

- How about Selection Sort ?
  - What is its running time in terms of n?
  - Can we write Selection Sort in C ?