CS 5319
Advanced Discrete Structure

Lecture 16:
Introduction to Automata Theory

Outline

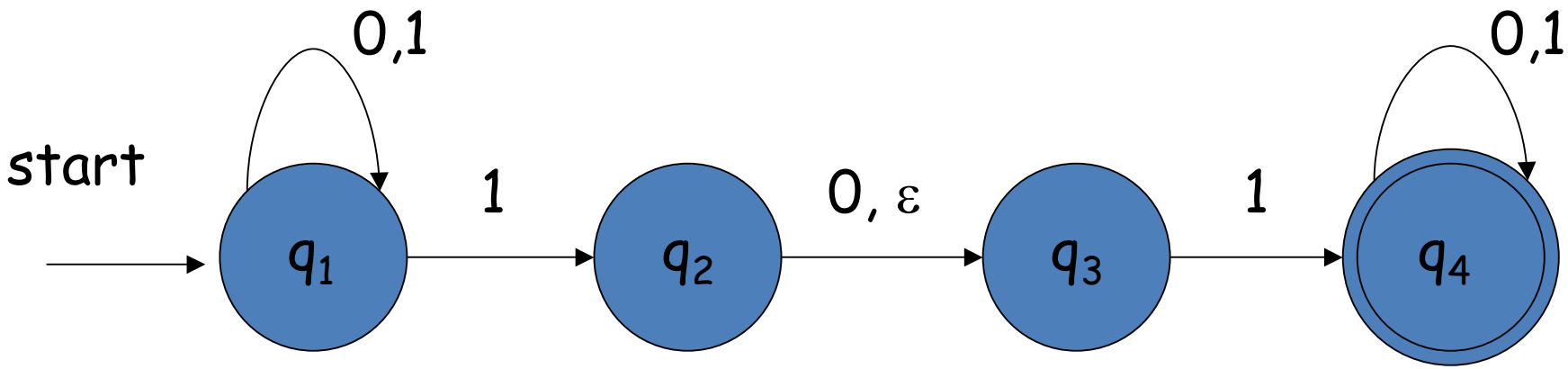
- Deterministic Finite Automaton (DFA)
- Pumping Lemma
- Non-Deterministic Finite Automaton (NFA)
- Equivalence between DFA and NFA

Non-Deterministic Finite Automaton

Non-Deterministic Finite Automaton

- When processing a string in a DFA, there is always a unique state to go next when each character is read
  - It is because for each state in DFA, there is exactly one state that corresponds to each character being read
- In an NFA, several choice (or no choice) may exist for the next state

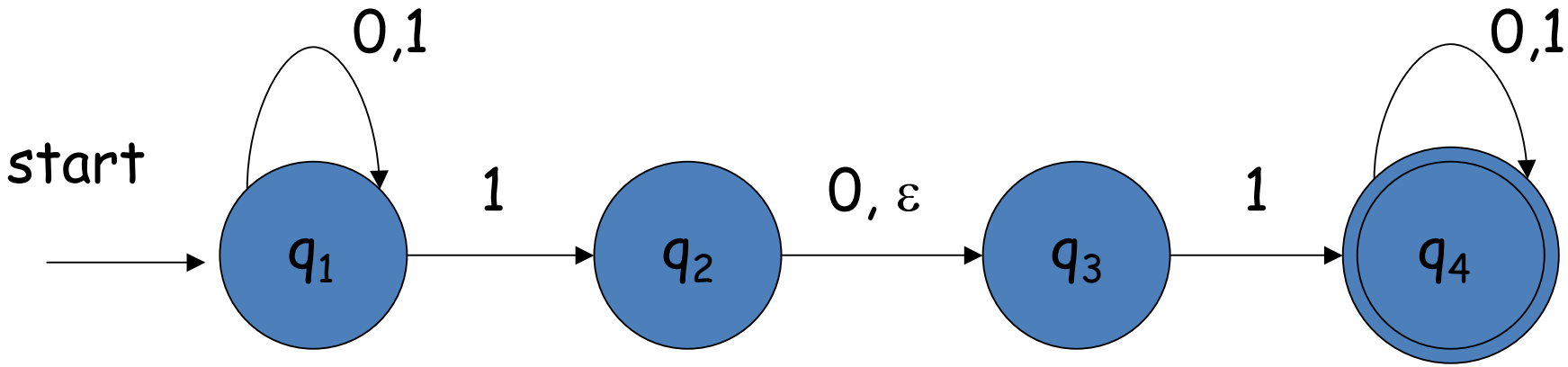
Example of NFA



Difference with DFA

- Can move to more than 1 states, or nowhere
- Can move to a state without reading anything

Example of NFA



- An NFA accepts a string *w* if there is a way to land to an accepting state after reading *w*

  Ex : The above NFA accepts 11, 0101, 0011, …

  but rejects 001, 100, 00100, …

Equivalence Between DFA and NFA

Equivalence between DFA and NFA

- Suppose *L* is a language of a certain DFA
  - Immediately, we can see that *L* is a language of some NFA
- A more interesting result is as follows :

Theorem (DFA has same power as NFA) :

> Suppose *L* is a language of a certain NFA.
>
> Then there is a DFA such that *L* is its language.

Equivalence between DFA and NFA

- Proof Idea :

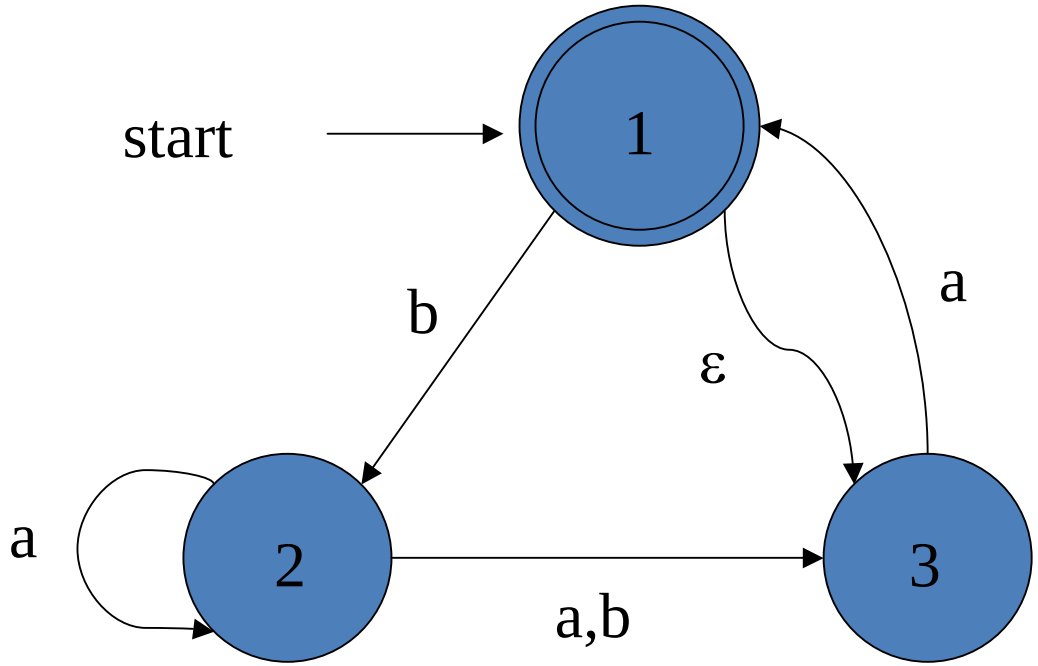
  Let N = given NFA, with states Q_N

  We shall construct a DFA D, with states Q_D,

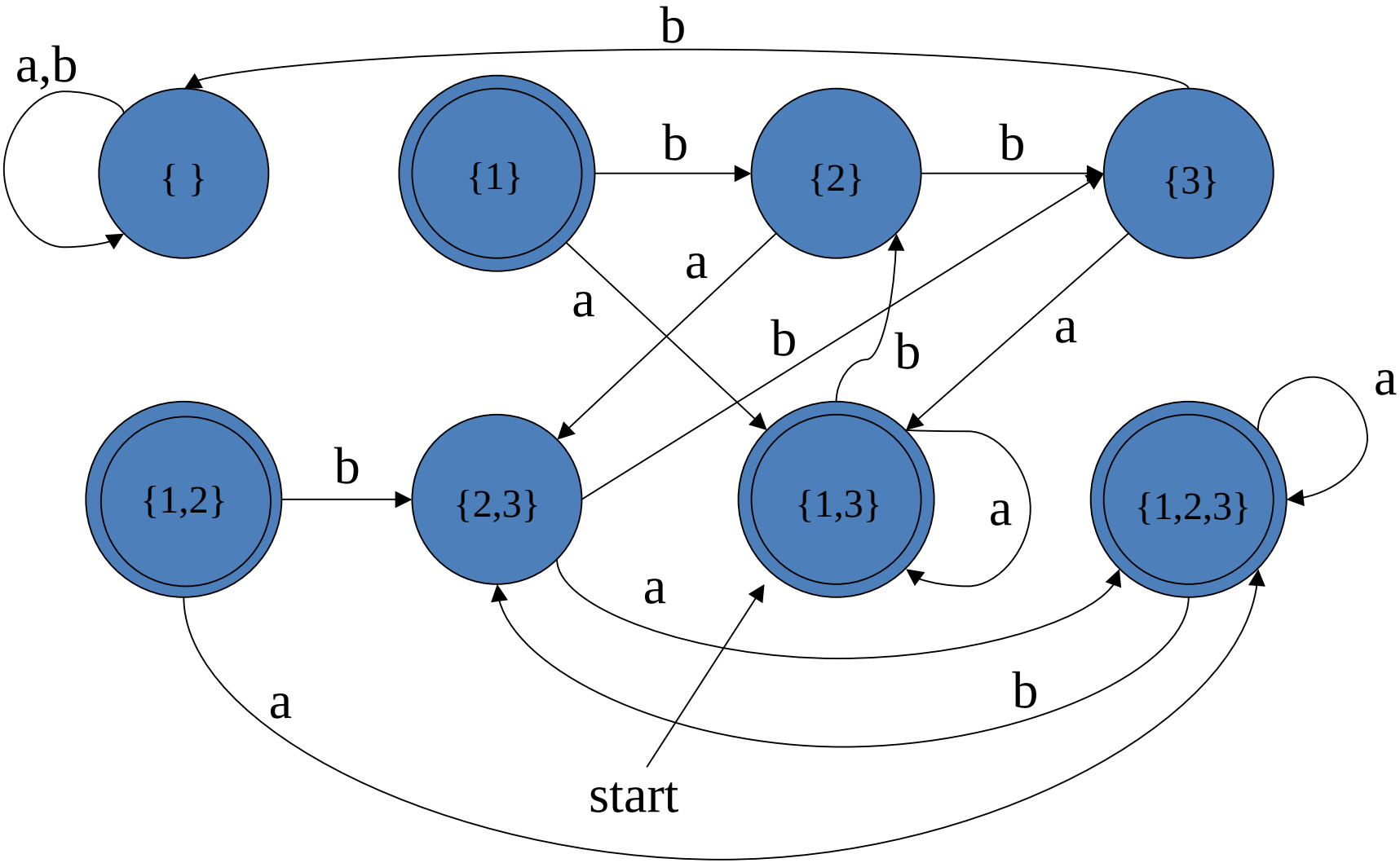
  such that each state in Q_D corresponds to a particular subset of states in Q_N

  Then, we design the transitions (arrows) in D such that we can simulate the reading in N by the reading in D

Constructing a DFA from NFA



Constructing a DFA from NFA



Properties of Regular Language

Theorem :

> Suppose A and B are regular languages.
>
> Then $A \cup B$ is also regular.

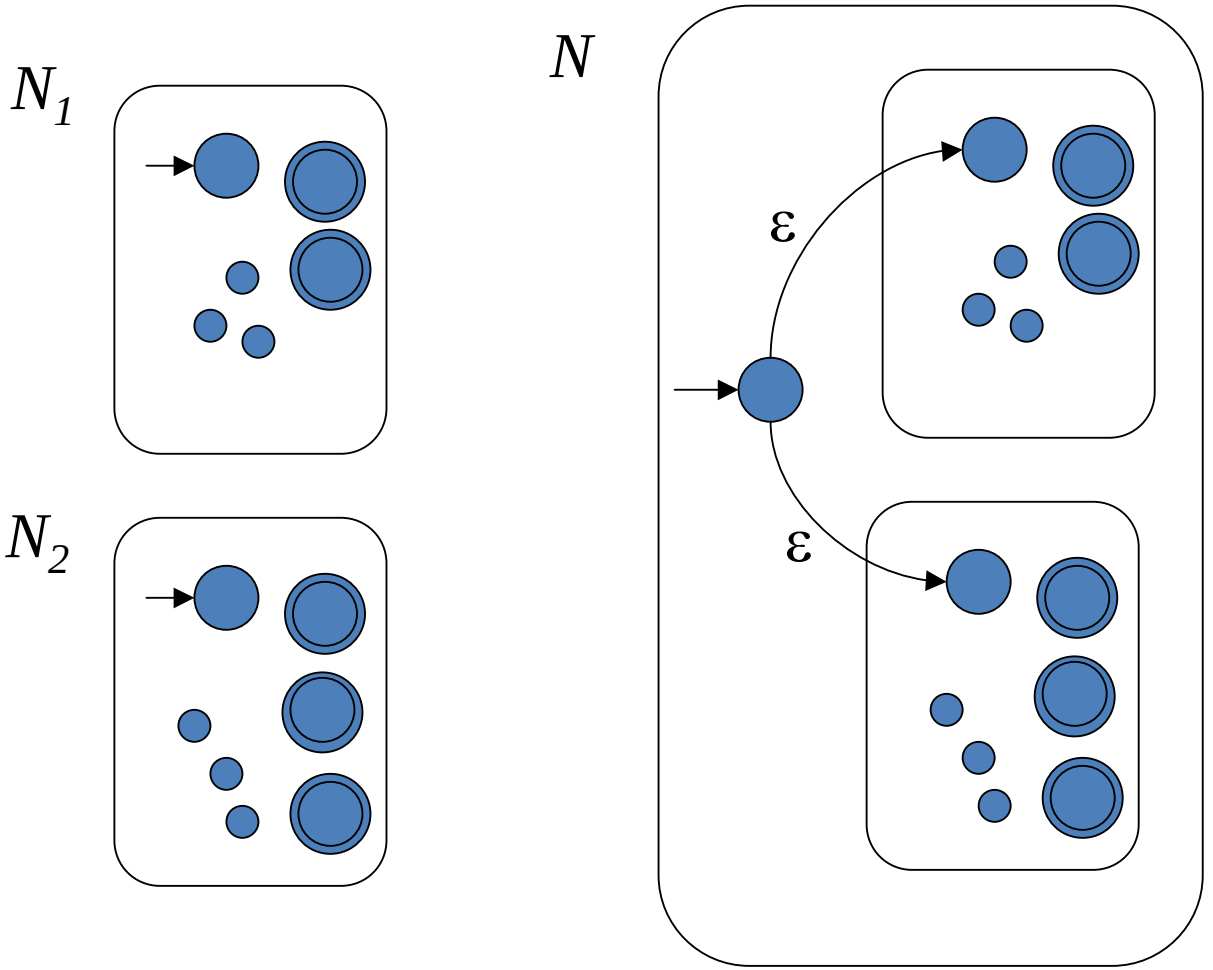
Proof Idea:

Construct NFA N with language $A \cup B$.

Let N_1 = NFA with language A

N_2 = NFA with language B

Proof



Properties of Regular Language

Theorem :

> Suppose A and B are regular languages. Then $A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$ is also regular.

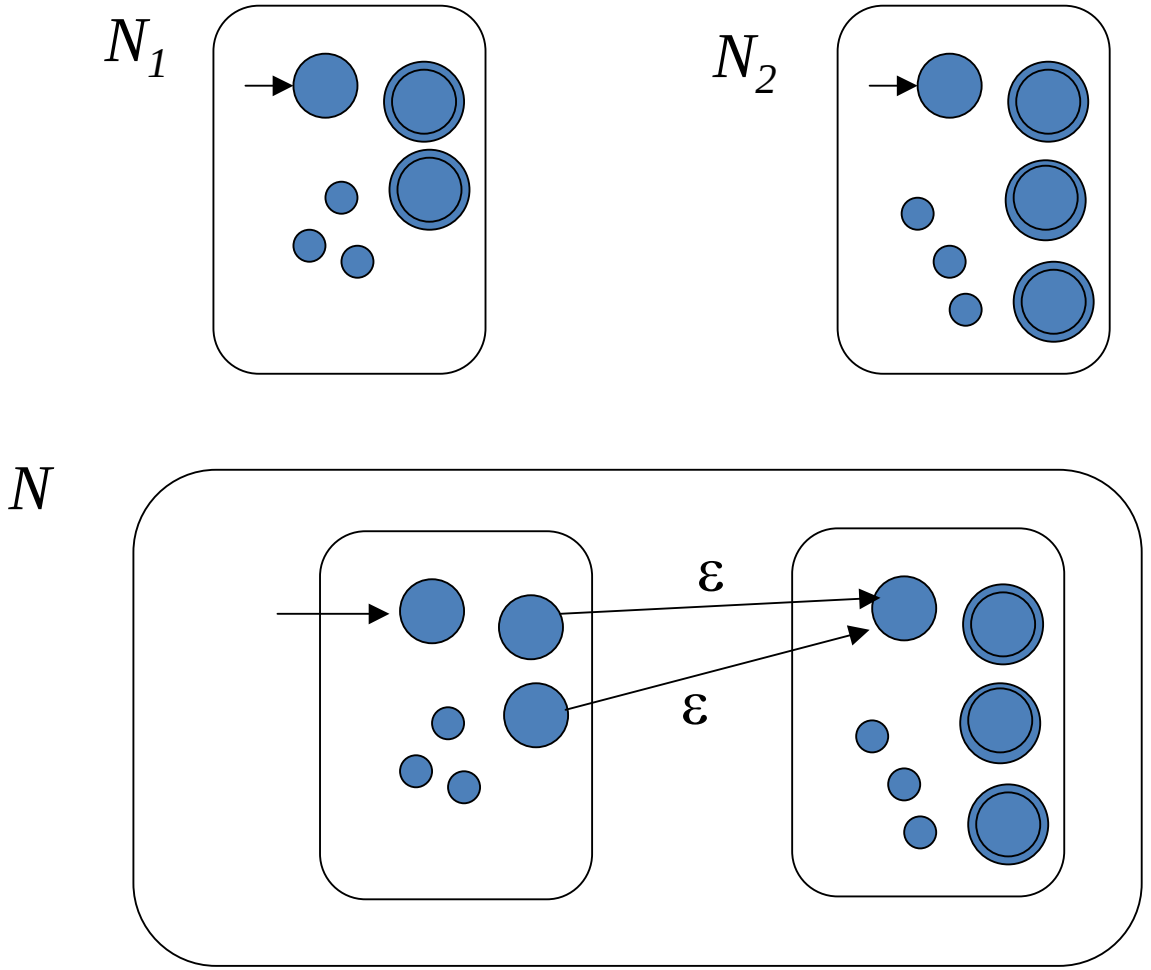
Proof Idea:

Construct NFA N with language $A \circ B$.

Let N_1 = NFA with language A

N_2 = NFA with language B

Proof



Properties of Regular Language

Theorem :

Suppose A is regular. Then the language

$$A^* = \{ x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and } x_i \in A \}$$

is also regular.

Proof Idea:

Construct NFA N with language A^* .

Let N_1 = NFA with language A

Proof