Let $L$ be a regular language.

Define $L^{\text{REV}}$ to be the language

$$L^{\text{REV}} = \{ S \mid S \text{ is the reverse of some string in } L \}$$

Show that $L^{\text{REV}}$ is regular.

**EX:** If “001” is in $L$, “100” is in $L^{\text{REV}}$. 
**Question 1**

*Hint*: Given the DFA for $L$, show that it can be modified to an NFA for $L^{\text{REV}}$. To describe your idea, please use the following DFA as an example (where the leftmost state is the start state).
Question 2

Design a DFA that accepts the language with 
\[ \Sigma = \{0,1\} : \\
\{S \mid \text{the number of } 01^{'} \text{'s occurrences in } S = \\
\text{the number of } 10^{'} \text{'s occurrences in } S \} \\
\]

**EX:** 001110, 110111001101
Question 3

Show that the language
\[ \{ 1^x \mid x \text{ is prime} \} \]
is non-regular.

*Hint*: Use pumping lemma.
A palindrome is a string that can be read forward and backward in the same way. For example, "00100" and "010010" are palindromes.
Question 4

Prove that the language \{S \mid S \text{ is a palindrome}\} is non-regular.

**Hint**: Use pumping lemma.
Question 5 (Challenge)

Let

$$\Sigma_3 = \begin{bmatrix} [0], [0], [0], [1], \ldots, [1] \end{bmatrix}$$

$\Sigma_3$ contains all size 3 columns of 0s and 1s. A string in $\Sigma_3$ gives three rows of 0s and 1s.
Question 5 (Challenge)

Consider each row to be a binary number and let $B = \{ \omega \in \Sigma_3^* \mid \text{the bottom row of } \omega \text{ is the sum of top two rows} \}$

For example,

\[
\begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}
\in B \quad \text{but} \quad
\begin{bmatrix}
1 & 0 \\
0 & 0 \\
1 & 1
\end{bmatrix}
\notin B.
\]
Question 5 (Challenge)

Show that $B$ is regular.

(*Hint*: Working with $B^{\text{REV}}$ is easier. You may assume the result claimed in question 1.)
Let $L_1$ and $L_2$ be two regular languages.

Prove that $L_1 \cap L_2$ is also regular.

*Hint*: For any regular language, we can build a DFA that accepts it.