Advanced Discrete Structure
Homework 7 Tutorial

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Question 1

Let L be a regular language.

Define L^{REV} to be the language

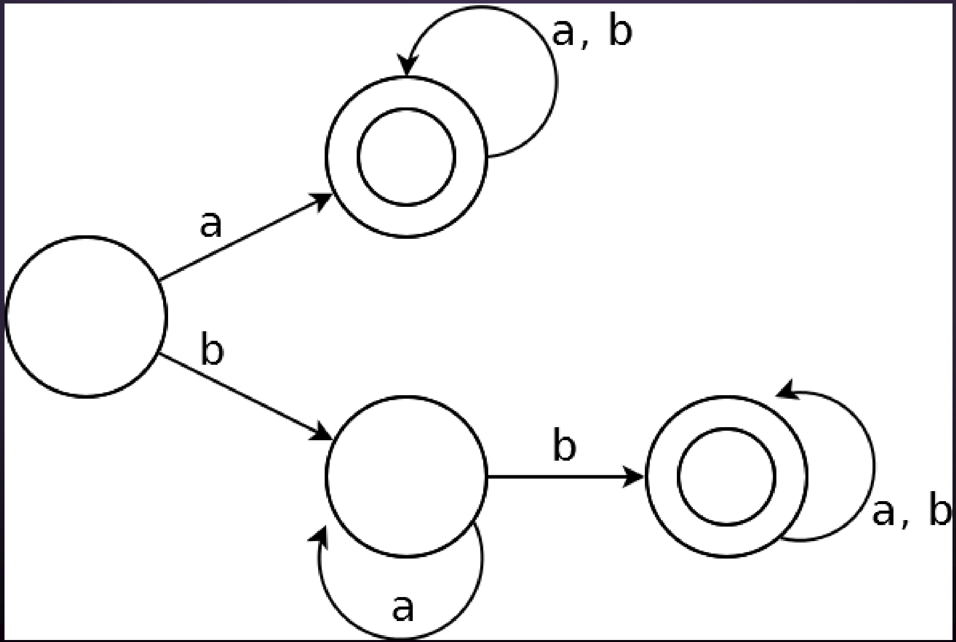
$$L^{\mathrm{REV}} = \{\, S \mid \mathrm{S\ is\ the\ reverse\ of\ some\ string\ in\ L} \,\}$$

Show that L^{REV} is regular.

EX: If "001" is in L, "100" is in L^{REV}.

Question 1

Hint: Given the DFA for L, show that it can be modified to an NFA for L^{REV}. To describe your idea, please use the following DFA as an example (where the leftmost state is the start state).



Question 2

Design a DFA that accepts the language with $\Sigma = \{0,1\}$:

$$\{S \mid \text{the number of } 01's \text{ occurrences in } S = \text{the number of } 10's \text{ occurrences in } S\}$$

EX: 001110, 110111001101

Question 3

Show that the language

$$\{\, 1^x \mid x \text{ is prime} \,\}$$

is non-regular.

Hint: Use pumping lemma.

Question 4

A palindrome is a string that can be read forward and backward in the same way.

For example, "00100" and "010010" are palindromes.

Question 4

Prove that the language $\{S \mid S \text{ is a palidrome}\}$ is non-regular.

Hint: Use pumping lemma.

Question 5 (Challenge)

Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \cdots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Σ_3 contains all size 3 columns of 0s and 1s.

A string in Σ_3 gives three rows of 0s and 1s.

Question 5 (Challenge)

Consider each row to be a binary number and let $B = \{\omega \in \Sigma_3^* \mid \text{the bottom row of } \omega \text{ is the sum of top two rows}\}$

For example,

$$\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in B \text{ but } \begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} \notin B.$$

Question 5 (Challenge)

Show that B is regular.

(*Hint*: Working with B^{REV} is easier. You may assume the result claimed in question 1.)

Question 6 (Challenge)

Let L_1 and L_2 be two regular languages.

Prove that $L_1 \cap L_2$ is also regular.

Hint: For any regular language, we can build a DFA that accepts it.