1. Let $L$ be a regular language. Define $L^{\text{REV}}$ to be the language

$$L^{\text{REV}} = \{ S \mid S \text{ is the reverse of some string in } L \}.$$ 

Show that $L^{\text{REV}}$ is regular.

*Hint:* Given the DFA for $L$, show that it can be modified to an NFA for $L^{\text{REV}}$. To describe your idea, please use the following DFA as an example (where the leftmost state is the start state).

```
state a
  a -> b
  b -> a

state b
  a -> a
  b -> b
```

2. Design a DFA for the language with $\Sigma = \{0, 1\}$:

$$\{S \mid \text{the number of 01’s occurrences in } S = \text{the number of 10’s occurrences in } S\}.$$ 

3. Show that the language $\{1^x \mid x \text{ is prime}\}$ is non-regular.

4. A palindrome is a string that can be read forward and backward in the same way. For example, “00100” and “010010” are palindromes. Prove that the language $\{S \mid S \text{ is a palindrome}\}$ is non-regular.

5. (Challenging: No marks) Let $\Sigma_3$ contains all size-3 columns of 0s and 1s as follows:

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$ 

A string in $\Sigma_3$ gives three rows of 0s and 1s. Consider each row to be a binary number. Let

$$B = \{ \omega \in \Sigma_3^* \mid \text{the bottom row of } \omega \text{ is the sum of the top two rows} \}.$$ 

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B,$$

but

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \not\in B.$$ 

Show that $B$ is regular.

*Hint:* Working with $B^{\text{REV}}$ is easier. You may assume the result claimed in Question 1.

6. (Challenging: No marks) Let $L_1$ and $L_2$ be two regular languages. Prove that their intersection, $L_1 \cap L_2$, is also regular.