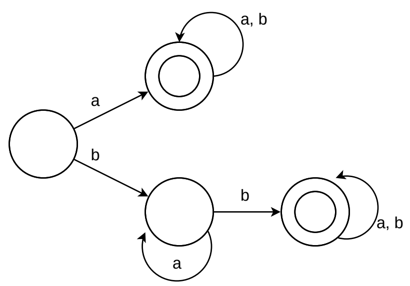
CS5319 Advanced Discrete Structure

Homework 7

Due: 3:20 pm, January 05, 2012 (before class)

1. Let L be a regular language. Define L^{REV} to be the language
$$L^{\mathrm{REV}} = \{S \mid S \text{ is the reverse of some string in } L\}.$$
Show that L^{REV} is regular.
Hint: Given the DFA for L, show that it can be modified to an NFA for L^{REV}. To describe your idea, please use the following DFA as an example (where the leftmost state is the start state).


2. Design a DFA for the language with $\Sigma = \{0, 1\}$:
$$\{S \mid \text{the number of 01's occurrences in } S = \text{the number of 10's occurrences in } S\}.$$

3. Show that the language $\{1^x \mid x \text{ is prime}\}$ is non-regular.

4. A palindrome is a string that can be read forward and backward in the same way. For example, "00100" and "010010" are palindromes. Prove that the language $\{S \mid S \text{ is a palindrome}\}$ is non-regular.

5. (Challenging: No marks) Let Σ_3 contains all size-3 columns of 0s and 1s as follows:
$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ldots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$
A string in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number. Let
$$B = \{\omega \in \Sigma_3^* \mid \text{the bottom row of } \omega \text{ is the sum of the top two rows}\}.$$
For example,
$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \text{ but } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$
Show that B is regular.
Hint: Working with B^{REV} is easier. You may assume the result claimed in Question 1.

6. (Challenging: No marks) Let L_1 and L_2 be two regular languages. Prove that their intersection, $L_1 \cap L_2$, is also regular.