Advanced Discrete Structure Homework 6 Tutorial

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Question 1

Let $(A,*)$ be a semigroup.

Show that, for a, b, c in A, if $a * c = c * a$ and $b * c = c * b$, then $(a * b) * c = c * (a * b)$.

Question 1

Notice that $(A,*)$ is a semigroup.

Things seem to be usually true may be not true here.

Question 2

Let $(A,*)$ be a monoid such that for every x in A, $x * x = e$, where e is the identity element. Show that $(A,*)$ is an abelian group.

Question 2

Like question 1, things seem to be usually true may not be true here.

Question 3

Let $(H,\cdot)$ and $(K,\cdot)$ be subgroups of a group $(G,\cdot)$. Let

$$HK = \{h \cdot k \mid h \in H, k \in K\}$$

Show that $(HK,\cdot)$ is a subgroup if and only if $HK = KH$.

Question 4

The *order* of an element a in a group is denoted to be the least positive integer m such that $a^m = e$, where e is the identity element. (If no positive power of a equals e, the order of a is denoted to be infinite.) Show that, in a finite group, the order of an element divides the order of the group.

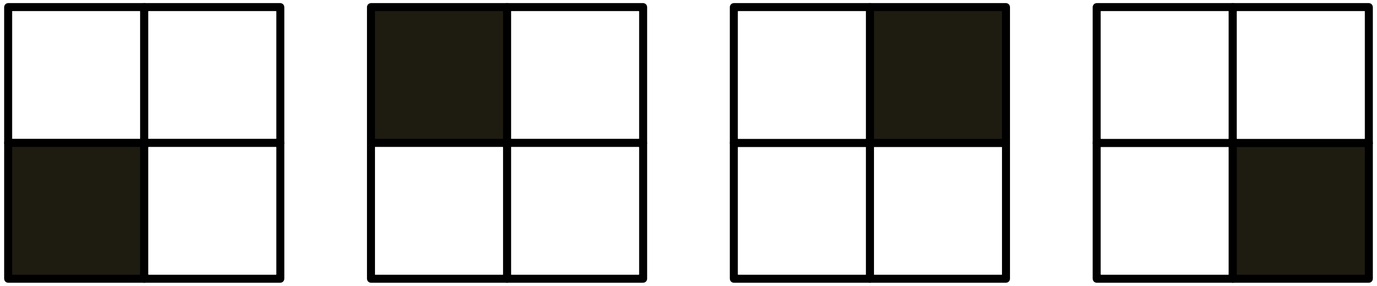
Question 5&6

You can use Burnside's Theorem

in the following questions.

Question 5(a)

Determine the number of distinct 2×2 chessboards whose cells are painted white and black. Two chessboards are considered distinct if one cannot be obtained from another through rotation.



Question 5(b)

Repeat part (a) for 4×4 chessboards.

Question 6

Consider a cube with each face colored by one of the n colors. In how many distinct ways can the cube be colored?

(Two colorings are equal if one can be transformed to the other by rotating the cube.)

Hint: There are 24 kinds of rotations!